Section calculation for reinforced concrete members subjected to combined bending, shear and torsion

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ABSTRACT

In concrete design, due to the complex behaviour of concrete members in the cracked state, the load bearing capacities due to bending, shear and torsion are often considered separately and the results of the capacity checks for all individual load actions are somehow combined at the end. This approach is generally accepted because of its simplification, but it has certain weaknesses because the interaction between the actions of different load components is not well reflected in the calculation model. The result obtained can be very conservative or very uncertain depending on the design case. For complex sections such as box or hollow sections in bridge structures, the shear walls may behave very differently depending on the load actions there. This paper presents an improvement of the MNVT shear model for reinforced concrete members under combined bending, shear and torsional loading, focusing on the modelling and determination of the shear flows in the cracked cross section. In addition to the theoretical background of the shear model, selected examples are given. Some comments on design are also provided.

RESUMEN

En el cálculo del hormigón, debido al complejo comportamiento de los elementos de hormigón en estado fisurado, las capacidades portantes debidas a flexión, cortante y torsión suelen considerarse por separado y los resultados de las comprobaciones de capacidad para todas las acciones de carga individuales se combinan de algún modo al final. Este enfoque suele aceptarse por su simplificación, pero presenta ciertos puntos débiles, ya que la interacción entre las acciones de los distintos componentes de la carga no queda bien reflejada en el modelo de cálculo. El resultado obtenido puede ser muy conservador o muy incierto dependiendo del caso de diseño. En el caso de secciones complejas, como los cajones o las secciones huecas de las estructuras de puentes, las paredes de cortante pueden comportarse de forma muy diferente en función de las acciones de las cargas que se ejerzan sobre ellos. Este artículo presenta una mejora del modelo de cortante MNVT para elementos de hormigón armado sometidos a cargas combinadas de flexión, cortante y torsión, centrándose en la modelización y determinación de los flujos de cortante en la sección transversal fisurada. Además de los fundamentos teóricos del modelo de cortante, se muestran algunos ejemplos. También se ofrecen algunos comentarios sobre el dimensionamiento.

KEYWORDS: calculation model, shear reinforcement, concrete, torsion, shear strength, strut-and-tie model, structural design

PALABRAS CLAVE: modelo de cálculo, armadura de cortante, hormigón, torsión, resistencia a cortante, modelo de bielas y tirantes, diseño estructura.

1. Introduction

Concrete members are often subjected to multiple load components simultaneously. For frame elements, these include normal force, bending moments, shear forces, and torsional moments. Each of these components, when dominant, can lead to a specific failure mode. As a result, determining the exact failure mode for members under combined loads is a complex challenge. In practice, two primary failure modes are typically considered: *bending* and *shear*.

Due to the complexity of calculations, a practical design approach involves assessing bending and shear capacities separately and then combining the results to evaluate the total load-bearing capacity. This method is widely accepted in civil engineering since it allows for calculations to be performed manually or with simple computational tools.

It is well known that an interaction exists between bending and shear, and this effect must be considered in design. Shear forces can introduce additional normal forces, which are redistributed within both the concrete and reinforcement along the member axis. Fully accounting for this effect across all six force components in a cross-section requires an advanced numerical approach. However, the development of such methods often depends on individual research groups or software companies, leading to a lack of consistency and a universally accepted approach. Current design codes also provide limited guidance on accurate calculation and verification methods for concrete members subjected to combined loads, resulting in persistent uncertainties - particularly regarding shear design.

Very recently, the new generation of Eurocode 2 [1] has provided a more detailed description of the thin-walled cross-section method, which explicitly allows the consideration of combined loads.

This paper presents an improvement of the MNVT shear model developed by Elmer [2] for reinforced concrete members under combined load, with a focus on modelling and determining shear flow in cross-section. As it is intended for the design, the shear model is formulated in accordance with Eurocode 2 [1]. The proposed theory is illustrated through selected examples that can be solved manually as well as using the software FAGUS [3], developed by Cubus AG.

2. Fundamentals of design concept for combinations actions

2.1. General

In the new generation of Eurocode 2 [1], design of members subjected to combined actions is defined in Section 8.3 *Torsion and combined actions*, in which the modelling method to determine the shear force in sub-sections and the design procedure for combination of actions are clearly described. This is a big advantage compared to the current Eurocode 2 [4].

Two approaches for considering the interaction of combined actions are proposed in the new Eurocode 2: a *simplified approach* and a concept based on *detailed analysis*, which are briefly described as below.

Interaction formula

In EC2 new generation [1] a simplified and conservative verification of the resistance of cross sections subjected to combination of internal forces is proposed based on a linear criterion:

$$\sum\left(\frac{S_{\mathrm{Ed}}}{S_{\mathrm{Rd}}}\right)_i \leq 1 \tag{1}$$

in which the S_{Ed}/S_{Rd}-ratios describe different actions including shear, torsion and bending and the corresponding bending moments. Two separate verifications can be carried out considering the following combinations:

a) bending, torsion and axial forces; and
b) shear, torsion and axial force

Thin-walled section

Alternatively, from the interaction formula, the design procedure for combination of actions can be based on a thin-walled closed section, in which the sectional forces and moments are replaced by a statically equivalent set of normal- and shear stress distributions. The distribution of normal and shear forces may be determined by conventional elastic or plastic methods. The calculation of reinforcement and check of compression in concrete struts of each individual wall can be performed using formulas for membrane elements or general shear design [1].

To illustrate this approach, a calculation method commonly used for analysing the stiffening system of buildings is presented. The fundamental calculations for a system composed of orthogonal shear walls are presented below.

The centre of gravity of the section is determined according to [5] as follows:

$$y_s = \frac{\sum\left(I_{iy} y_i\right)}{\sum I_{iy}} \tag{2}$$

$$z_s = \frac{\sum\left(I_{iz} z_i\right)}{\sum I_{iz}} \tag{3}$$

The load parts due to displacement are calculated as:

$$\overleftarrow{H}_{y,i} = \frac{H_{y,M} \cdot EI_{z,i}}{\sum_{1}^{n} EI_{z,i}} \tag{4}$$

$$\overleftarrow{H}_{z,i} = \frac{H_{z,M} \cdot EI_{y,i}}{\sum_{1}^{n} EI_{y,i}} \tag{5}$$

The load parts due to rotation are:

$$H_{y,i} = -\frac{M_{x,M} \cdot EI_{z,i} \cdot z_{Mmi}}{\sum_{1}^{n} EI_{\omega}} \tag{6}$$

$$H_{y,i} = +\frac{M_{x,M} \cdot EI_{y,i} \cdot y_{Mmi}}{\sum_{1}^{n} EI_{\omega}} \tag{7}$$

The shear force in each direction is then determined as the sum of both components - one due to displacement and the other due to rotation.

It is important to note that the above equations are valid for orthogonal walls only. For more general cases a modification of these equations or another method is required.

2.2. Shear design

The shear capacities calculated on the basis of the contributions of shear reinforcement and concrete strut are determined according to [1] as follows:

$$\tau_{Rd,sy} = \rho_w \cdot f_{ywd} \cdot \left(\cot\theta + \cot\alpha\right) \cdot \sin\alpha \tag{8}$$

$$\tau_{Rd,max} = \nu \cdot f_{cd} \cdot \frac{\cot\theta + \cot\alpha}{1 + \cot^2\theta} \tag{9}$$

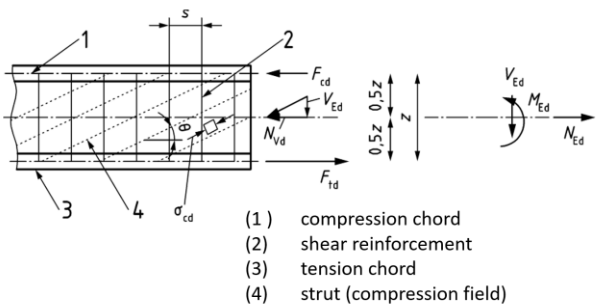

Figure 1. Shear model [1]

For reinforced concrete members with inclined shear reinforcement (45° ≤ α), the following is defined:

$$\tan\frac{\alpha}{2} \le \cot\theta \le \cot\theta_{min} \tag{10}$$

where the inclination angle α of shear reinforcement is typically limited by 90°.

The capacity of concrete strut is influenced by the cracking state of concrete, which is described through the reduction factor ν of concrete compressive strength. This reduction factor can be determined using Eq. (11), in which ε_x is the average concrete strain of the considered concrete region.

$$\nu = \frac{1}{1 + 110\left(\varepsilon_x + \left(\varepsilon_x + 0.001\right)\cot^2\theta\right)} \le 1 \tag{11}$$

3. Approach to section analysis

3.1 General

To have an economical design while ensuring required structural safety, the approach to directly determine the internal forces in load-bearing components of the section under consideration of combined actions is used.

The shear flow in the cross section is first determined by modelling the cross section through a substitute system composed of shear walls, in which the shear forces and torsional moment are considered. For this purpose, the portion of shear force in each shear wall is determined depending on the relative stiffness of the wall to that of the whole shear wall system. The interaction between bending and shear as well as torsion is described through the additional normal force and bending moment induced from the shear forces in the shear walls. The bending analysis is then carried out with the effective bending moment and normal force, which include the additional forces due to shear forces and torsional moment. The approach to section analysis is summarized as below:

- *Modelling the cross section through substitute system and determining the shear forces in shear walls*
- *Taking the additional forces induced from shear forces and torsional moment into account, calculating the cross section under bending and normal force*
- *Verification of shear capacity in each shear wall*
- *Verification of bending capacity with effective bending moment and normal force M^* & N^*.*

Since the shear capacity depends on the inclination angle θ of the concrete strut, which varies depending on the longitudinal force in the shear wall, the section calculation is generally carried out iteratively.

3.2 Determination of shear flow

Under combined load action, internal normal and shear forces are distributed to all structural elements within the considered section. By treating the shear reinforcement and surrounding concrete as a shear wall system, the section can be represented as a set of shear walls, which may be either connected or separate, depending on the reinforcement detailing and applied loads.

To determine the shear forces in these shear walls, a small finite element model can be used, where each shear wall is represented as a spring aligned with its respective direction. The stiffness of each spring is set equal to the stiffness of the corresponding shear wall and is proportional to the moment of inertia of the wall.

In the general case, there are *n* unknown internal forces to be solved for three action quantities (V_y, V_z, and T) [3], as shown in Fig. 2.

The influence of shear forces on the axial forces and bending moments is directly incorporated into the bending analysis by introducing an additional "*internal*" tensile axial force and bending moments. The effective load is determined as follows:

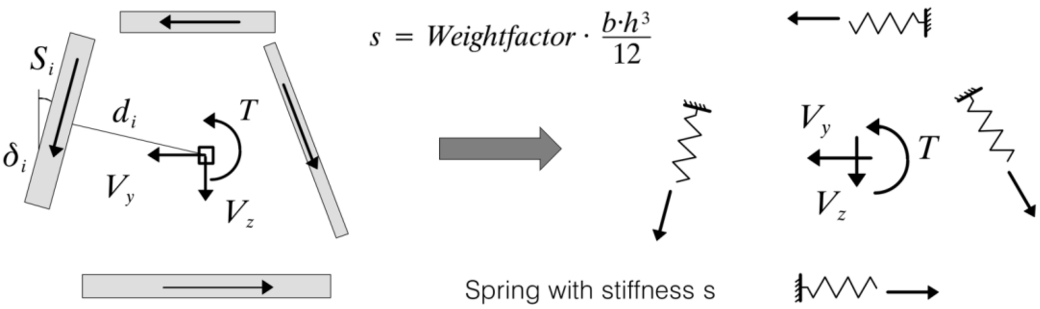

Figure 2. Thin-walled section and calculation model [3]

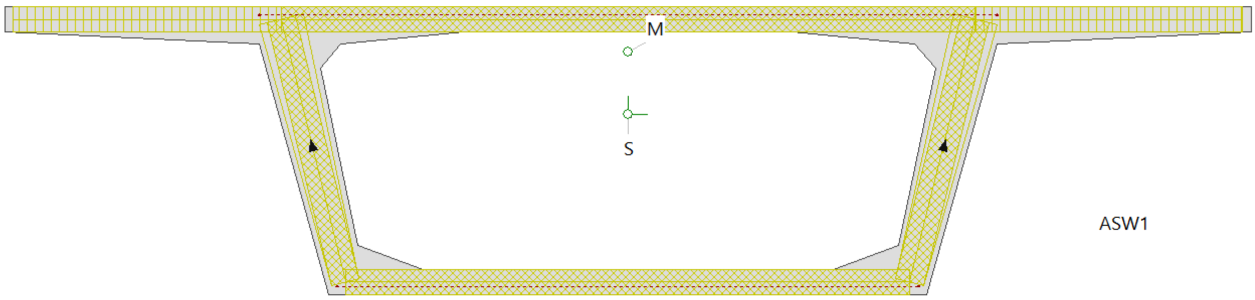

Figure 3. Shear walls in the FAGUS program

$$F_{\mathrm{si}} = S_{\mathrm{i}}\left(\cot\theta - \cot\alpha\right) \tag{12}$$

$$N^{*} = N + \sum F_{\mathrm{si}} \tag{13}$$

$$M_{\mathrm{y}}^{*} = M_{\mathrm{y}} + \sum F_{\mathrm{si}} \cdot \Delta z_{\mathrm{si}} \tag{14}$$

$$M_{\mathrm{z}}^{*} = M_{\mathrm{z}} + \sum F_{\mathrm{si}} \cdot \Delta y_{\mathrm{si}} \tag{15}$$

with F_{si} is the normal force contribution of the i$^{\mathrm{th}}$ shear wall and Δz_{si}, Δy_{si} are the distances from middle of shear wall to the centre of gravity (point **S**) of the cross section or to the reference point of the cross section, which is used as the loading point for the section, see Fig. 3.

3.3 Verification of load-bearing capacity

In the section analysis with combined load actions, the capacity of the section can be considered as insufficient if one of the following failure modes occurs:

Bending capacity

- capacity of concrete in compression zone is insufficient, concrete compressive strain exceeds the designed ultimate strain
- capacity of longitudinal reinforcement is insufficient, strain in the reinforcement exceeds the designed ultimate strain.

Shear capacity in walls

- capacity of concrete strut is insufficient
- capacity of shear reinforcement is insufficient, strain in the shear reinforcement exceeds the designed ultimate strain.

Shear capacity in connection zones between walls

- capacity of concrete strut is insufficient
- capacity of transverse reinforcement in related walls is insufficient.

It should be noted that the connection between the shear walls and the compression belts as well as the tension belts is important part to ensure the load transfer between elements and therefore needs to be carefully checked. The verification of these connections requires forces at two cross-sections of the considered reinforced concrete segment.

For each shear wall, it is necessary to define the effective depth of the *truss system* (strut and tie). This depends on the effective lever arm of the cross-section and the detailing of reinforcement, especially the anchorage of shear reinforcement.

3.2.1 Capacity verification of shear walls

The load-bearing capacities of shear walls can be verified using the classical *strut and tie model* and Eq. (8) & (9) presented in Section 2.2.

Alternatively, shear walls can also be considered as membrane elements and the capacity verification is carried out using the *compression stress fields*. Based on the equilibrium of stress, the required reinforcement and the required compressive strength of concrete strut are determined as below, see also Fig. 4:

$$\sigma_{\mathrm{Edy}} + \frac{\left|\tau_{\mathrm{Edxy}}\right|}{\cot\theta + \cot\alpha} \le \rho_{\mathrm{y}} \sin\alpha \cdot f_{\mathrm{yd,y}} \tag{16}$$

$$\sigma_{\mathrm{Edx}} + \left|\tau_{\mathrm{Edxy}}\right|\left(\cot\theta - \cot\alpha\right) \le \rho_{\mathrm{x}} \cdot f_{\mathrm{yd,x}} \tag{17}$$

$$\sigma_{\mathrm{cd}} = \left|\tau_{\mathrm{Edxy}}\right| \frac{1+\cot^{2}\theta}{\cot\theta + \cot\alpha} \le \nu \cdot f_{\mathrm{cd}} \tag{18}$$

Here, the reduction factor ν of the concrete compressive strength in Eq. (18) can be determined according to Eq. (11).

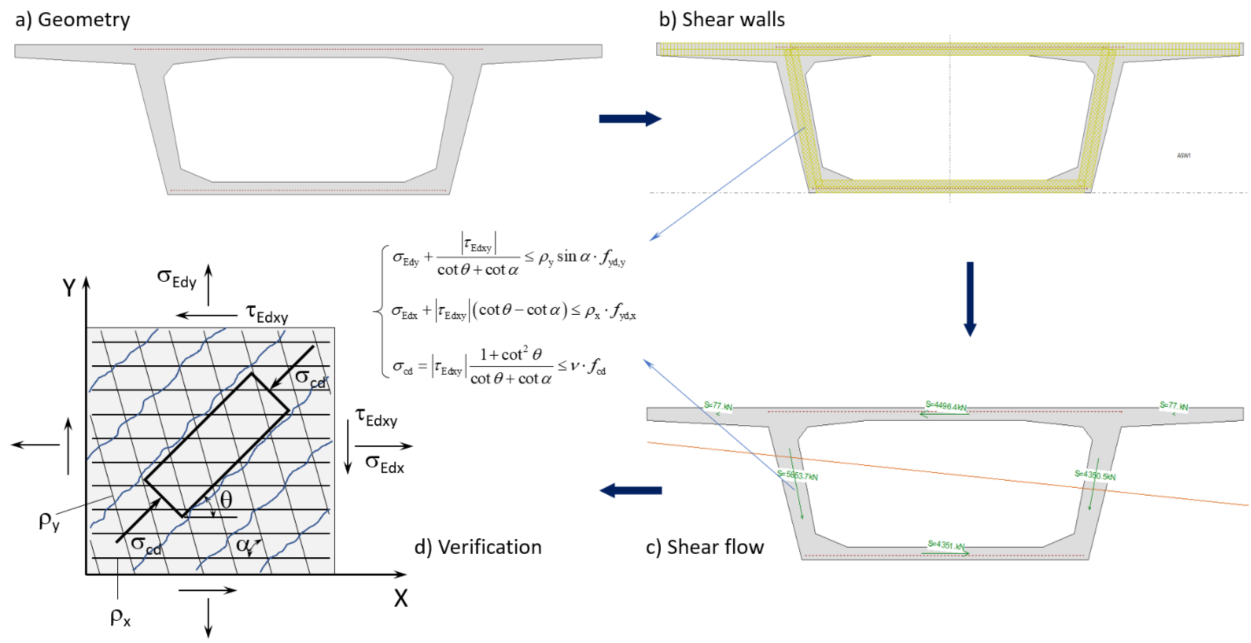

Figure 4. Shear walls

3.2.2 Capacity verification of connection zones

For sections composed of some shear walls, such as T-section or hollow section, the connection zone between the walls (e.g. the webs and the flanges) can be critical and thus should be designed and checked with care.

An example is the West Seattle bridge, in which many cracks occurred in the box girders. This shows problems of load transfer between the bottom flange and the webs as well as of the provided longitudinal and transverse shear reinforcement in the concrete box.

Figure 5. West Seattle Bridge *(source: https://www.westsideseattle.com)*

The verification of the shear capacity at the connection between webs and flanges is clearly presented in EC2 [1], see the calculation model in Fig. 6. For this verification, two cross sections are required to determine the shear force on the section cut between webs and flanges.

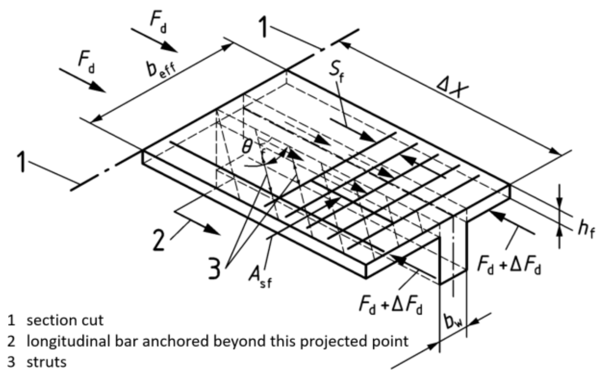

Figure 6. Connection between flange and web [1]

4. Examples

4.1 Shear walls

To illustrate and validate the calculation method, an example of building stiffening system taken from [5] is used. The system is subjected to horizontal forces in both directions H_y = -100 kN and H_z = -100 kN.

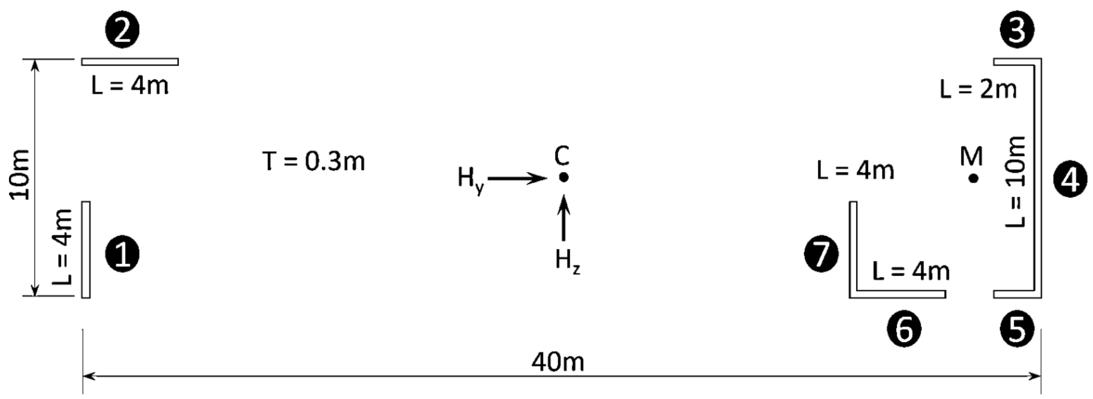

Figure 7. Shear walls of stiffening system

The system is modelled and the shear flow distribution is determined using two approaches: a) the calculation method (**M1**) presented in Section 3.2. and b) the calculation method (**M2**) using Eq. (2)-(7) presented in Section 2.1. The shear walls are numbered from W1 to W7 as shown in Fig. 7. The calculation results are summarized in Table 1 and Table 2, respectively.

Table 1. Shear forces in walls (acc. to M1, *FAGUS*)

Wall	L (m)	S (kN)	V_Y (kN)	V_Z (kN)	T (kNm)
W1	4	46.5	-0.1	-46.5	-1260.7
W2	4	49.7	-49.7	0.1	-277.0
W3	2	6.2	-6.2	0.0	-35.0
W4	10	-42.4	0.0	-42.4	532.1
W5	2	-4.9	-4.9	0.0	20.0
W6	4	39.1	-39.1	0.0	159.1
W7	4	-11.3	0.0	-11.3	55.0
Sum of forces			-100	-100	-806.5

Table 2. Shear forces in walls (acc. to M2, *by hand*)

Wall	$H_{y,i}$ (kN)	$H_{z,i}$ (kN)	$H_{Hy,i}$ (kN)	$H_{Hz,i}$ (kN)	$H_{Ry,i}$ (kN)	$H_{Rz,i}$ (kN)
W1	0	-46.4	0	-5.7	0	-40.7
W2	-49.8	0	-44.4	0	-5.3	0
W3	-6.2	0	-5.6	0	-0.7	0
W4	0	-42.5	0	-88.7	0	46.2
W5	-4.9	0	-5.6	0	0.7	0
W6	-39.1	0	-44.4	0	5.3	0
W7	0	-11.2	0	-5.7	0	-5.5
Sum	-100	-100				

It can be seen that the shear forces in the walls calculated using the two methods are very close to each other, confirming the accuracy of the implemented finite element method in the calculation of shear forces in shear walls.

4.2 Hollow section

A column with a hollow section is investigated, see Fig. 8. The load action is a combination of normal force N = -50 000 kN, bending moment M_y = 100 000 kNm, shear force V_z = 10 000 kN and torsional moment T = 15 000 kNm.

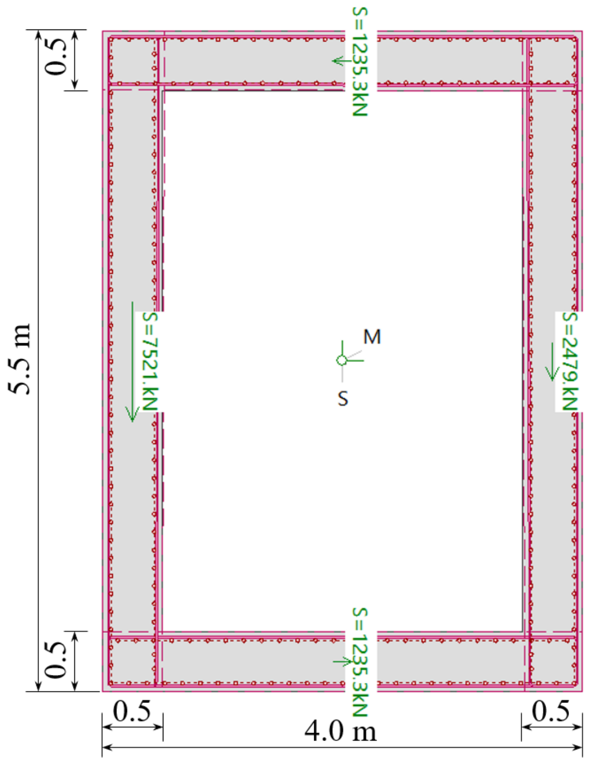

Figure 8. Shear forces in shear walls

To verify the shear capacity of section, considering the contributions of both shear reinforcement and concrete strut, the inclination angle θ of the concrete strut is selected with cotθ = 1.2, corresponding to θ = 39.8°, as a simplified estimate, which is recommended according to DIN EN 1992-2/NA [4] for members without tensile force. The shear reinforcement is assumed to be perpendicular to the member axis (α = 90°). The calculated shear forces in shear walls due to the given combined shear force and torsion are presented in Fig. 8.

Interaction between shear, torsion and bending

As presented in the Section 3, the interaction between the different force components (shear force, torsional and bending moments) is described through the modified load actions, in which the longitudinal forces induced from shear forces in shear walls are taken into account. For the given case, the modified load is calculated as below:

$$N^{*} = -50000 + \begin{pmatrix} 1235.3 + 1235.3 \\ + 7521 + 2479 \end{pmatrix} \cdot 1.2 \\ = -3535.3\,\text{kN} \tag{19}$$

$$M_y^{*} = (1235.3 - 1235.3) \cdot 2.5 \cdot 1.2 = 0 \tag{20}$$

$$M_z^{*} = (7521 - 2479) \cdot 1.75 \cdot 1.2 \\ = 10588.2\,\text{kNm} \tag{21}$$

The longitudinal stresses and strains of the cross section are then calculated using the effective forces determined above, which is illustrated below using program FAGUS [3].

Action forces

No.	AP	P	Bending and axial force N [kN]	Bending and axial force M_y [kNm]	Bending and axial force M_z [kNm]	Shear forces and torsion V_y [kN]	Shear forces and torsion V_z [kN]	Shear forces and torsion T [kNm]
1	!ULS	S	-50000.0	100000.0	0	0	10000.0	15000.0
		*	-35035.3	100000.0	10588.2	0	10000.0	15000.0

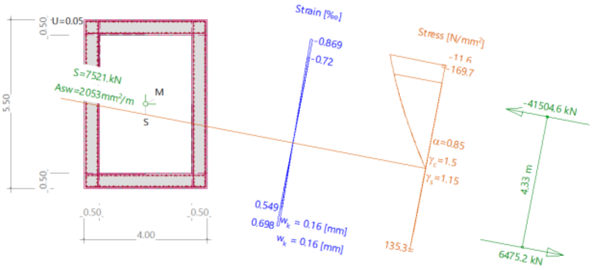

Figure 9. Bending calculation

Interestingly, although no bending moment M_z is applied in this example, the neutral axis of the cross-section is inclined relative to the y-axis of the coordinate system. This inclination results from the interaction between shear force, torsion, and bending - an effect that could not be observed if bending and shear analyses were performed separately.

For design, pre-defined longitudinal and shear reinforcement is often used. If the inclination angles of the concrete strut θ and the shear reinforcement α are selected, the shear forces in the shear walls and their corresponding longitudinal forces can be determined based purely on the applied loads. This allows for a straightforward calculation of the required longitudinal and shear reinforcement. In cases where the reinforcement is fixed, and the load-bearing capacity is verified for different load scenarios, the method of varying the inclination angle θ of concrete strut can be employed to optimize reinforcement and account for the differing behaviour of each shear wall. Depending on the applied design codes, limiting values for θ must be considered in the capacity verification of reinforced concrete member.

The additional longitudinal force induced by shear forces in shear walls is highly dependent on both the inclination angle θ of the concrete strut and the inclination angle α of the shear reinforcement, as expressed in Eq. (12). A smaller θ combined with a larger α results in a greater tensile force due to shear effect.

The inclination angle θ of the concrete strut can also be determined using the *Simplified Modified Compression Field Theory* (SMCFT), developed by Bentz et al. [6], as follows:

$$\theta = (29° + 7000\varepsilon_x)\left(0.88 + \frac{s_{xe}}{2500}\right) \leq 75° \tag{22}$$

in which ε_x is the average strain of cross section and s_{xe} effective spacing of shear reinforcement. Such estimate enables consistent consideration of shear walls and rapid convergence of the calculation of the entire cross-section.

Influence of inclination angle of shear reinforcement

To examine the influence of the inclination angle α of shear reinforcement on the load-bearing capacity, three values of α are chosen for illustration: 90°, 75° and 115°, see Fig. 10.

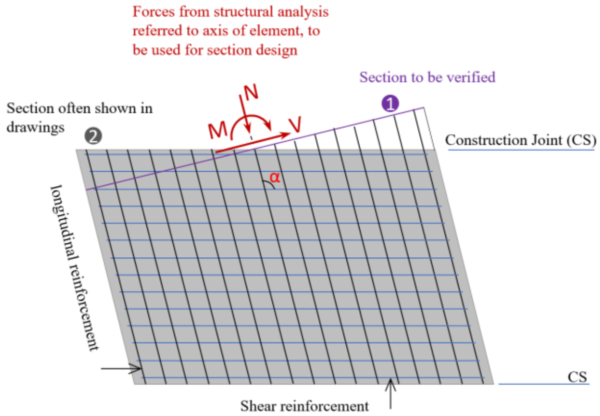

Figure 10. Shear resistance of inclined members

The shear reinforcement determined for the case of α = 90° is used as reference for comparison. Given the applied load, the demand-to-capacity (D/C) ratio is calculated and presented in the table below:

Table 3. Load bearing capacity of section

Wall	α = 75°	α = 90°	α = 105°
Load-bearing capacity (D/C)	0.85	1.00	**1.34**

The calculation result shows that the inclination angle α of shear reinforcement has a significant impact on shear capacity. The effect is particularly critical for cases α > 90°, which is also seen in the practice, e.g. in bridge structures, where inclined concrete members are frequently to be found.

5. Summary

This paper discusses the design and analysis of reinforced concrete sections subjected to combined load, including axial force, bending moments, shear forces, and torsional moments. The traditional approach is to evaluate the bending and shear forces separately before combining them to assess the overall load-bearing capacity. However, this method neglects the interaction between bending and shear.

Although the latest generation of Eurocode 2 provides more detailed guidelines on the thin-walled section method, the complexity of calculations and inconsistencies in design standards still lead to uncertainties, particularly in shear capacity assessment.

An improved MNVT shear model is presented to analyse shear stress distribution in reinforced concrete cross sections. Conventional calculation methods and the FAGUS software are used to illustrate this approach.

The paper also examines the influence of the inclination angle α of shear reinforcement and θ of concrete struts on optimizing structural performance and reinforcement design. The results indicate that the inclination angle of shear reinforcement significantly affects shear capacity, especially when α > 90°, which is sometimes the critical case in bridge structures.

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