Tendon optimization for prestressed concrete structures using genetic algorithms

Optimización de tendones para estructuras de hormigón postensado mediante algoritmos genéticos

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RESUMEN

La contribución presenta un código informático que optimiza la fuerza y geometría de un tendón de postensado. La parte mecánica realiza un análisis dependiente del tiempo de la estructura de hormigón postensado, teniendo en cuenta la retracción, la fluencia, la relajación del tendón, la fricción y el deslizamiento del anclaje, todos ellos componentes necesarios de pérdidas de postensado a corto y largo plazo. La parte de optimización ejecuta un algoritmo genético con parámetros elegidos de la geometría del tendón sometidos a optimización con respecto a las restricciones mecánicas y geométricas. La variable minimizada es la fuerza de pretensado. En la última sección se presenta un estudio paramétrico de un tablero de puente postensado de 3 vanos.

ABSTRACT

The contribution presents a computer code that optimizes a prestress tendon force and geometry. The mechanical part computes time-dependent analysis of the prestress concrete structure taking into account shrinkage, creep, tendon relaxation, friction and anchor slip, i.e., all necessary components of prestress short- and long-term losses. The optimization part executes genetic algorithm with chosen parameters of the tendon geometry subjected to optimization with respect to mechanical and geometrical constraints. The minimized variable is prestressing force. A parametric study of a 3-span post-tensioned bridge deck is presented in the last section.

PALABRAS CLAVE: hormigón postensado, optimización del tendón, dependiente del tiempo, algoritmo genético

KEYWORDS: prestressed concrete, tendon optimization, time-dependent analysis, Genetic algorithm

1. Concept of prestressing

Prestressing is a special state of stress and deformation that is actively introduced into a structure to improve its structural behavior. It allows to design lighter and more slender structures in a straightforward manner. Long span reinforced concrete girders and bridges can't be realized technically and economically without prestressing, among other things due to large displacements and difficult reinforcement arrangements.

Prestressing was originally designed to balance the effect of permanent loads and partially also variable loads – partial prestressing. Initiation of cracks was allowed since they close immediately after the variable load is removed, assuming a compressive reserve in the entire structure in long term. However, according to EN 1992-1-1 [1] the decompression limit for *frequent load combination* requires that no tensile stresses occur anywhere within a certain distance from the tendon or its duct. It requires higher prestress level compared to the former approaches (prior to EN); design of an optimal level of prestress may be challenging for statically indeterminate structures.

Optimal prestressing is not primarily based on the ultimate limit states but on a stress concept, where prestressing is sized for "design approaches" according to serviceability limit states with respect to the service life of the structure.

Design of prestressing is based on:

- state of stress;
- resistance of cross section (flexural, shear);
- deformations (during construction and throughout service life);
- method of construction, prestressing techniques;
- durability of the structure;
- economy.

1.1 Prestressing force and level of prestressing

Prestressing actively counterbalances external actions of permanent and variable loads. In the ideal case without any loss of prestress, the compressing force acts on cross sections of the beam as uniform stress from end to end. However, in the real design an engineer must consider variation of prestressing force due to losses from prestressing operations and time dependent losses due to creep and shrinkage of concrete and relaxation of steel.

The purpose of optimal prestressed structure design is to find optimal magnitude of prestressing force and optimal geometry of the tendon. Prestressing force in tendons and its layout should counterbalance external load and guarantee state of stress for minimal as well as for maximal loads. The compressive margin before the action of variable loads minimizes the tension in the final state of stress. Moreover, compression stresses must be below a certain threshold to avoid development of longitudinal cracks in concrete. No tensile stresses should be allowed under permanent load, although crack can develop under variable load. To prevent cracking in the surroundings of tendons, the tensile stress under *frequent load combination* is prohibited within a certain distance neighborhood of the tendon or its duct (recommended to be 100 mm for bridges). The exact conditions imposed on stress are stated in EN 1992-1-1 [1], chapter 7.2; conditions imposed on cracks are in chapter 7.3.

A higher level of prestressing should reduce the area of reinforcing steel since overall resistance of cross section is function of combination of prestressing and reinforcing steel. However too high level of prestressing can cause damaging due to dangerous cracking (during prestressing operation and also in service life) and inadmissible upward deflections of slender sections. High level of prestressing also causes large compression stresses which result into extensive creep.

Thus, it is obvious from the above that design of prestressing requires a comprehensive analysis.

2. Mechanical model

The mechanical model uses simple Euler-Bernoulli beam theory to compute deflections, internal forces, and stresses within the structure. Degrees of freedom are displacement and rotations at nodes. In every time step an inelastic steady-state analysis is performed by iterative search for global balance using elastic stiffness matrix of the structure. The necessity for inelastic analysis is given by time-dependent nature of the problem, i.e., by presence of creep, relaxation, shrinkage and aging of the structural parts.

2.1 Time-dependent analysis (TDA)

Two dimensional structural models composed of beam elements are analyzed in a fully time-dependent manner. The structural behavior is evaluated in time steps with exponentially growing size. Another discretization is applied spatially along the beam central lines, where internal forces and deflections are evaluated. The Euler-Bernoulli beam theory with linear stress and strain along the cross section is assumed. The integral approach to the time dependent analysis is implemented, the creep strain ε_c at every spatial, x, and time, t, point is evaluated:

$$\varepsilon_c(x,t) = \int_0^t \dot{\sigma}(t')J(t,t')dt' \qquad (1)$$

where $\dot{\sigma}$ is time derivative of normal stress, t' is dummy time variable, and J is the compliance function with aging [2]. The compliance function is evaluated according to EN 1992-2 [3]. Besides the creep strain, the shrinkage strain according to the same code is added.

Both short- and long-term losses of prestress force are implemented. The former group involves loss due to friction and anchor slip; the latter one takes into account relaxation of steel and elastic and inelastic deformation of concrete. The aging of concrete is considered via increase of its elastic modulus in time – see Fig. 1.

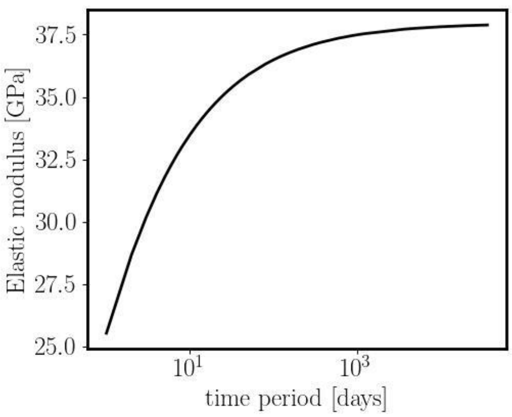

Figure 1. Development of concrete elastic modulus in time.

At each time step, an iterative procedure updating mechanical, creep and shrinkage strains and stresses in concrete as well as stresses in tendons takes place. The iterative loop starts with computing the concrete variables using the prestressing force from the last time step, change in structural boundary condition, aging, creep and shrinkage. All these calculations are done for the upper and bottom fibers of the beams. Then, the tendon force is updated based on relaxation and concrete normal strain. The concrete strain at the tendon location is linearly interpolated from the normal strain at the top and bottom fibers. The new tendon force then serves as an input for the next iteration. Typically, around 5 iterations suffice to reach practically zero difference between the input and output tendon forces.

2.2 Discretization

An idealization of the tendon into the curve line allows to compute its actions (friction and radial forces) on the concrete beam. The radial forces are computed from the second differentiation of the tendon *z* coordinate with respect to *x*. The tendon is actually discretized into straight sections – see Fig. 2, the radial forces are then simply computed from the balance of these sections.

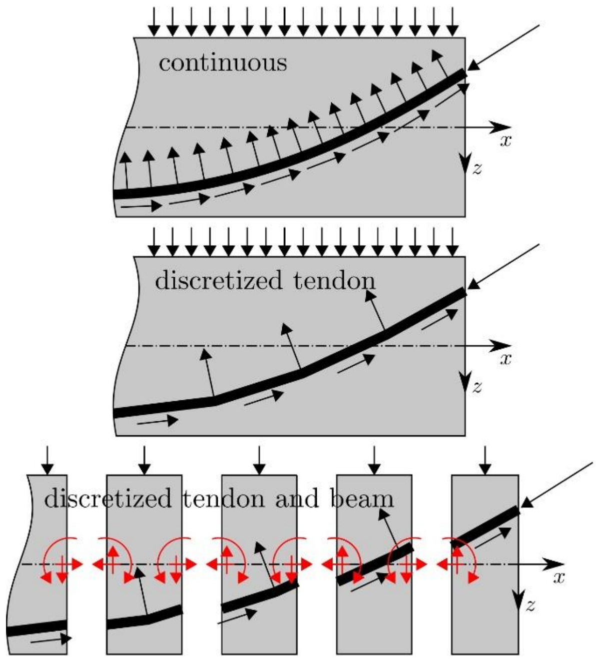

Figure 2. Actions on the beam due to external loads and tendon: continuous (top) and discretized idealizations (center and bottom). The red color indicates internal forces that balance all the external actions in black color.

An independent discretization of the beam is performed in parallel. The external actions within each beam section are included in the balance equations leading to internal forces (in red in Fig. 2).

3. Optimization

A genetic algorithm (GA) is a search and optimization technique inspired by the process of natural selection. It simulates evolution by iteratively improving a population of candidate solutions. Each solution, encoded as a chromosome, is evaluated based on a fitness function. Through genetic operators like selection (choosing the fittest individuals), crossover (combining traits of parent solutions), and mutation (randomly altering traits), the algorithm explores the solution space, gradually converging toward an optimal or near-optimal solution.

3.1 Definition of tendon geometry

To implement the developed optimization algorithm, the tendon's profile (geometry) must be parameterized. For this, a quadratic Bézier curve is used for each tendon region (typically a single span of a continuous beam) to represent the tendon's central line. The Bézier curve is convenient for shape optimization thanks to its straightforward shape control and versatility [4].

A quadratic Bézier curve is a smooth parametric curve defined by three control points: a start point C0, a control point C1, and an end point C2. The shape of the curve is influenced by C1, which acts as an attractor, pulling the curve toward itself but not necessarily lying on it. The curve starts at C0 and smoothly moves toward C2, with C1 influencing its trajectory. Quadratic Bézier curves always stay within the convex hull formed by the three control points, meaning they never stray beyond their defining triangle. They are also affine invariant, meaning that transformations like scaling, translation, and rotation affect the entire curve in a predictable way without distorting its fundamental shape. Because of their simplicity and computational efficiency, quadratic Bézier curves are widely used in vector graphics, font rendering, and computer-aided design.

For continuous beams, the connection between curves is achieved by incorporating an arch curve with matching tangential direction to the Bézier curves on both sides. The quadratic Bézier curve (which corresponds to a parabolic segment) is defined by three control points, each specified by its *x* and *z* coordinates.

Any point on the Bézier curve (tendon) can be calculated using the parameter t, where $t \in (0,1)$. The position, $\boldsymbol{x}_B = (x_B, z_B)$, of any point on the curve is determined using the parameter t through the following equation:

$$\mathbf{x}_{B(t)} = (1-t)^2\mathbf{C}_0 + 2(1-t)t\,\mathbf{C}_1 + t^2\mathbf{C}_2 \tag{2}$$

where $\boldsymbol{C}_i = (C_x^i, C_z^i)$ are control points coordinates.

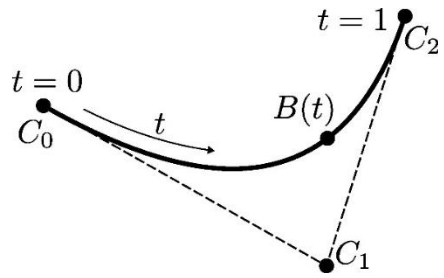

Figure 3. Quadratic Bézier curve.

For each individual in population, the vector of state variables (chromosome) is designed with the minimum possible number of variables to enable optimization without introducing an unnecessary complexity. For this reason, the horizontal coordinates of the control points C_x^i can be set as fixed. Since the geometry is defined by one Bézier curve per span of the beam, multiple curves are used in the case of a continuous beam, and only vertical coordinates of their control points are left free for optimization. The last remaining state variable is the prestressing force, a_P. The form of each individual's chromosome is therefore:

$$\mathbf{a} = \{C_z^1, C_z^2, \ldots, C_z^i, a_P\} \tag{3}$$

3.2 Fitness function

The fitness, $f_p(\mathbf{a})$, of each individual $\mathbf{a}$ is composed of the objective function, $f(\mathbf{a})$, and penalties for violating imposed constraints.

$$f_P(\mathbf{a}) = f(\mathbf{a}) + \sum_{i=1}^{n} q_i h_i(\mathbf{a}) \tag{4}$$

For the purposes of this optimization, only inequality constraints, h, are considered. It is worth noting that when the conditions are satisfied, the penalty function is equal to the objective function. The objective function $f(\mathbf{a})$ simply equals to the number of prestressing tendons (the lower the better) expressed as the prestressing force divided by the reference force per one tendon.

The constraints ensure that the concrete threshold stress values are not exceeded and that the tendon lies within the structure. The first two of the following equations address the minimum (maximum compression) and maximum (maximum tension) stresses in the beam, while the last two conditions ensure that the tendon geometry remains within the beam by constraining the geometry (the terms in the square brackets are indicator functions):

$$
\begin{aligned}
h_1 &: \int_0^l \left|\sigma_{x,\max}(x) - \sigma_{\max}\right| \; \left[\sigma_{x,\max}(x) > \sigma_{\max}\right] dx \\
h_2 &: \int_0^l \left|\sigma_{x,\min}(x) - \sigma_{\min}\right| \; \left[\sigma_{x,\min}(x) < \sigma_{\min}\right] dx \\
h_3 &: \int_0^l \left|z_B(x) - z_{\max}\right| \; \left[z_B(x) > z_{\max}\right] dx \\
h_4 &: \int_0^l \left|z_B(x) - z_{\min}\right| \; \left[z_B(x) < z_{\min}\right] dx
\end{aligned} \tag{5}
$$

An exterior penalty-based method known as the Automatic Dynamic Penalization (ADP) method was implemented to set up penalties, following the approach developed in Ref. [5]. The goal of this method is to allow the algorithm to explore the boundaries of non-feasibility. The evaluation of a non-feasible individual, with respect to each condition, is treated the same as the evaluation of the best individual in the generation. This approach is particularly useful for tendon geometry optimization, as the optimal solutions are expected to lie on the boundary of the infeasible region. To facilitate this search, non-feasible points are not eliminated during evaluation and are allowed to remain in the population, serving as attraction points.

According to ADP, the scaling coefficient for constraint violation, q_i, are determined separately for each generation based on the individuals that violate the i-th constraint:

$$
q_i = \max_{\mathbf{a} \in \mathbf{a}_{\mathrm{NF}i}} \left| \frac{f(\mathbf{a}_F^{\mathrm{BEST}}) - f(\mathbf{a})}{h_i(\mathbf{a})} \right| \tag{6}
$$

Here, a_F^{BEST} represents the best feasible individual in the generation. If no feasible individual is present in the generation, its value is assigned as zero.

Every time a new individual is assessed, an analysis of internal forces and stresses is conducted throughout the entire lifespan of the structure. A time dependent analysis (TDA) is performed, addressing creep, shrinkage, and prestress losses according to EN-1992 – see the previous Section 2. During the time dependent analysis, the maximum and minimum normal stresses for all times during the lifespan are recorded, and the envelope of extreme normal stresses throughout the lifespan is then used for evaluation of fitness of each individual.

3.3 Genetic operators

The selection process for individuals undergoing genetic operators differs between the first generation and subsequent generations. In the first generation, the total number of individuals is set to $2\,N_I = 1000$ individuals for a more robust screening of the configuration space. Selection is based solely on fitness evaluation, and 500 individuals are chosen to undergo crossover and mutation, proceeding to the second generation.

From the second generation onward, the number of individuals remains constant at $N_I = 500$. Selection is performed according to the following criteria: elitism is applied to feasible individuals according to each of $m = 4$ constraints. This results in the number of elite individuals being equal to $(m+1)N_E$. In this work, we set the number of elite individuals for each constraint, $N_E = 5$. The remaining $N_I - (m+1)N_E$ individuals are selected randomly. Elite individuals are included in the selection pool, meaning they may also be subjected to genetic operators. In such cases, both the original elite individual and its modified version are carried forward to the next generation. A detailed description of the crossover and mutation process is not the aim of this contribution, and we redirect the reader to [6].

4. Numerical example

In what follows, we present an example optimization of prestress tendon geometry and force for a three-span continuous beam representing a common bridge deck:

A) Solid slab, depth 1.2 m
B) Double T, depth 1.5 m
C) Voided slab, depth 1.2 m

4.1 Geometry and boundary conditions

The structure has a total length of 80.0 m consisting of spans 24 - 30 - 24 m and 1 m free ends. For slab deck is depth / span ratio 1.2 m / 30 m = 1/25; for double-T section is 1.5 m / 30 m = 1/20.

Figure 4 shows the three considered candidate cross-sections. The geometric properties of the cross-sections are listed in Table 1.

Table 1. Geometric properties of cross-sections

Property	Cross-section A	Cross-section B	Cross-section C
A [m²]	10.145	7.606	8.159
u [m]	14.436	17.600	14.436
Iy [m⁴]	1.250	1.424	1.147
z_{max} [m]	0.701	1.016	0.725
z_{min} [m]	-0.499	-0.484	-0.475

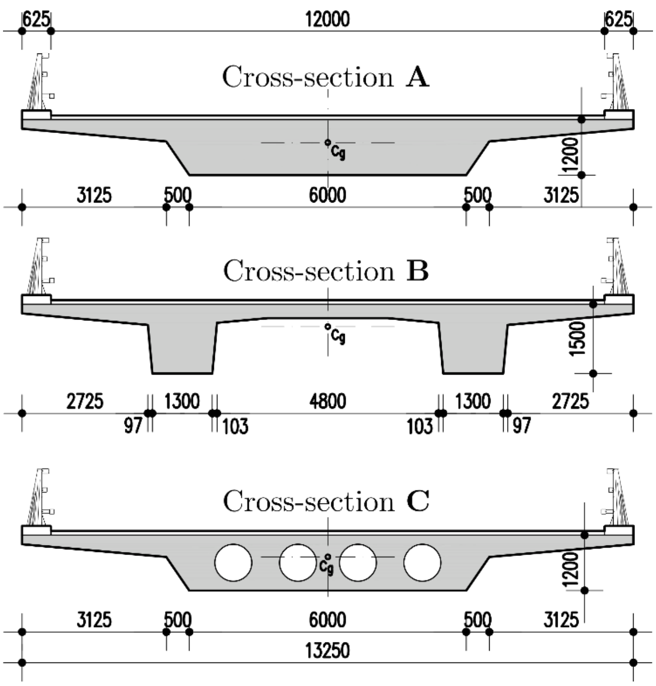

Figure 4. Cross-sections used for tendon optimization.

The model was loaded according to EN-1992-1-2. The used loading cases are listed in Table 2. In all cases, the characteristic load combination was the decisive one.

Table 2. Loading states

Load	Magnitude	Note
Self weight [kN/m]	253.7/178.2/ 203.9	Permanent, Start time: 7 days
Superimposed dead load [kN/m]	31.3	Permanent, Start time: 77 days
LM1 (TS) [kN]	2x 500	Traffic load
LM1 (UDL) [kN/m]	42	Traffic load
Temp. uniform [°C]	+26	Thermal load
Temp. uniform [°C]	-25	Thermal load
Temp. gradient [°C]	+15	Thermal load
Temp. gradient [°C]	-8	Thermal load

Table 3 lists material and construction parameters of a single tendon used during the optimization.

Table 3. Prestress tendon Y-1860 S7 parameters

Property	Value
Jacking stress [MPa]	1450
Prestressing time	8 days
Stressing order	Left, Right
Anchorage slip [m]	0.006
Tendon centre distance from surface [m]	0.150
Wobble coefficient	0.005
Friction coefficient	0.19

4.2 Optimization results

In what follows, we present the resulting tendon geometries optimized for each of the candidate cross-sections.

Figures 5, 6 and 7 display the optimization results for cross-section A, B and C, respectively. The first plot displays optimized tendon geometry. The second plot shows bending moments as obtained at the end of the service life, 100 years, originating from Permanent load (TDA: Permanent load), Tendon action (TDA: Tendon) and their combination (TDA). The third plot shows envelopes of normal stresses as recorded during the service life for Characteristic (Eq. 6.14), Frequent (Eq. 6.15) and Quasi-permanent load combinations (Eq. 6.16) according to EN 1990. Lastly, the fourth plot shows the force in tendon in the prestressing time (8 days) and 100 years.

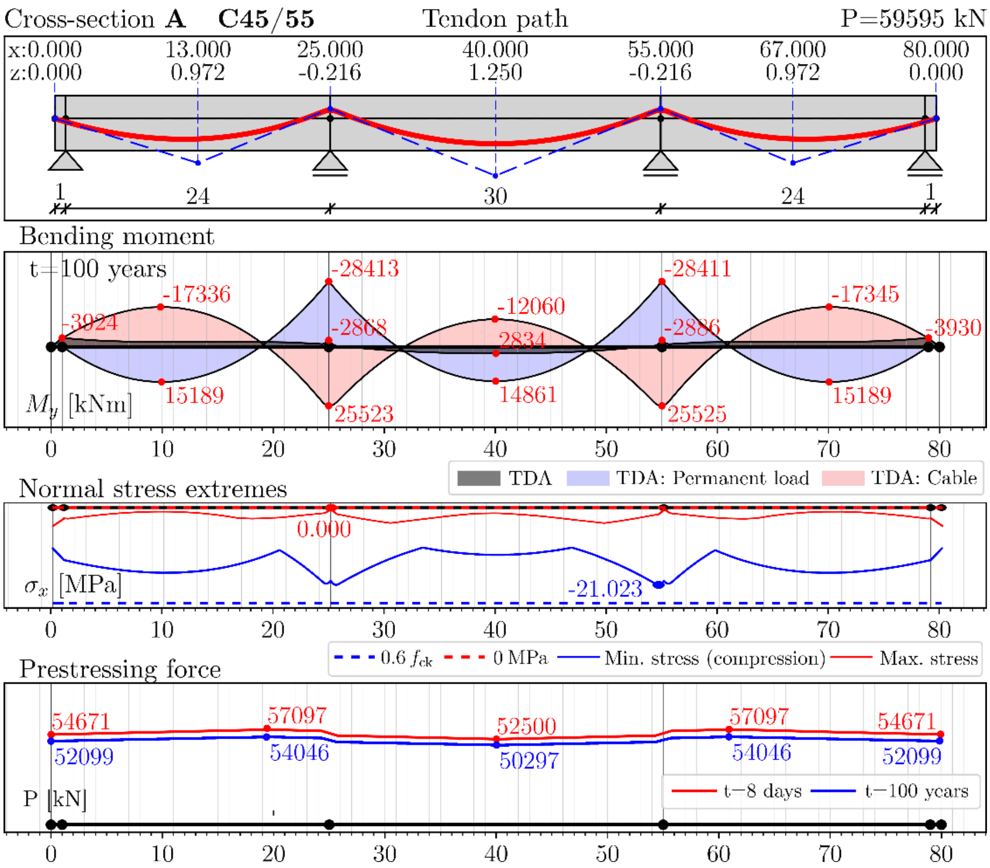

Figure 5. Optimization results: Cross-section A.

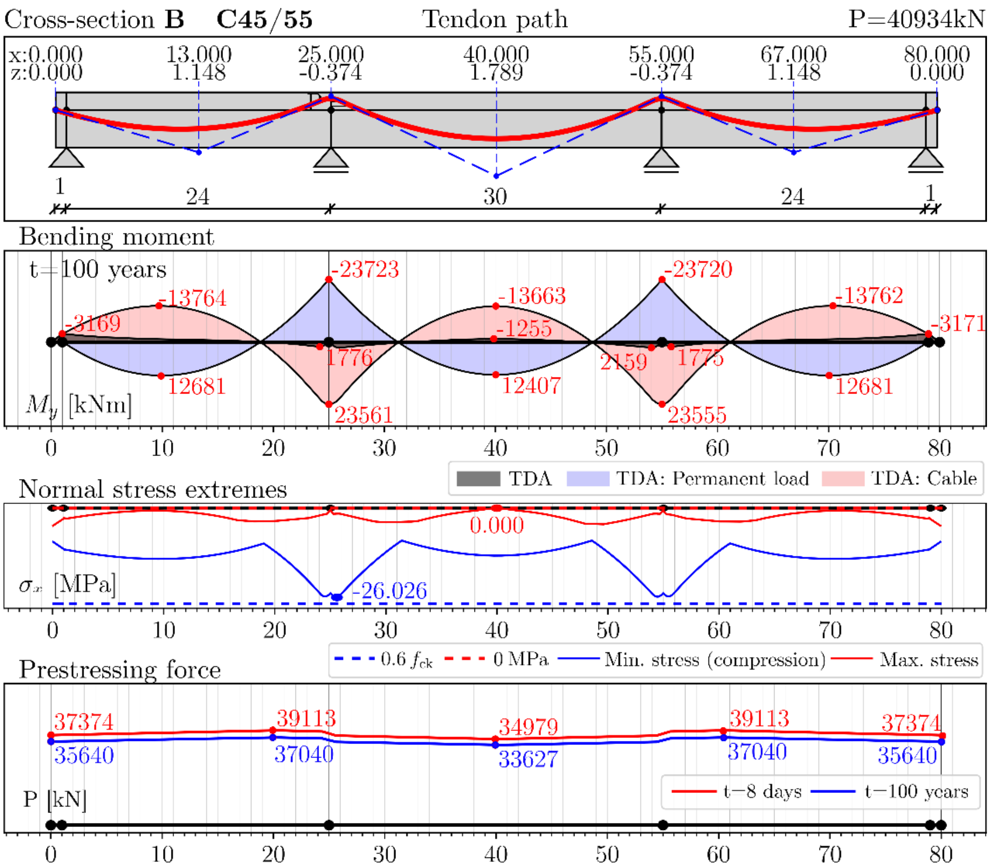

Figure 6. Optimization results: Cross-section B.

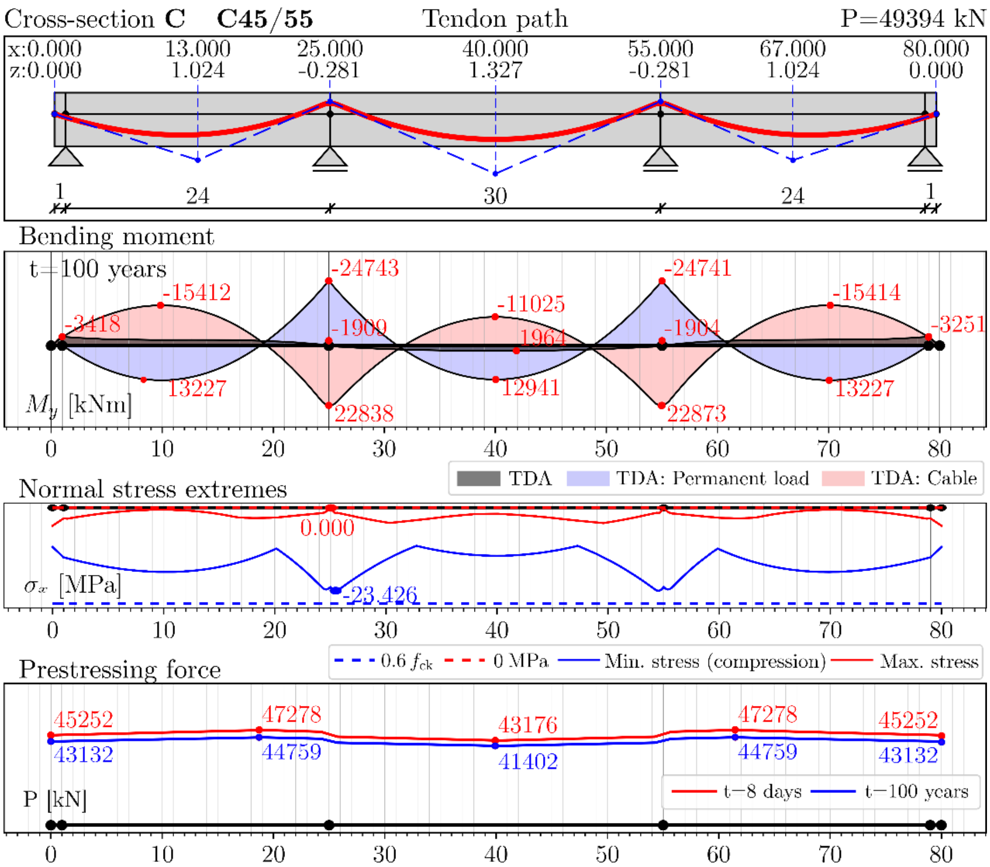

Figure 7. Optimization results: Cross-section C.

In all the cases the optimization found suitable tendon geometry satisfying all the given constraints. Cross-section C (voided slab) requires 21% less prestressing steel area compared to the solid slab of Cross-section A. The cross-section B (double T) requires the least prestressing, using 38% less prestressing steel compared to a solid slab.

As expected, decompression of the concrete for the Frequent load combination is the governing requirement for finding of minimum tendon force by the algorithm and the design of prestressed bridge deck, as shown by the red "Max Stress" curves, which touch zero stress in various zones.

In terms of stress limitation of concrete compression: *i)* Concrete compressive stress is close to the stress limitation 0.6 f_{ck} required for the Characteristic load combination only in the case of the double-T section, just above the support at the bottom fibers, due to the nature of the section; *ii)* For all three bridge deck cross sections, the stress limit of compression 0.45 f_{ck} for the Quasi permanent load is not decisive for the prestressed deck design.

Due to the lightweight deck cross-sections analysed in this parametric study the Characteristic load combination with traffic load (constraint of stress limitation 0.6 f_{ck}) has a greater role in the optimisation process than the Quasi-permanent load combination (constraint of stress limitation 0.45 f_{ck}).

The minimum prestressing force and tendon geometry remain unchanged regardless of the concrete class. The optimization algorithm found the same minimum prestressing force and tendon geometry for different concrete classes for a given cross section in parametric study. The results are presented for the minimum f_{ck} that satisfies stress limitations.

The execution time is approximately 2.5 hours per case on a standard PC equipped with an Intel Core i7-6700 processor running at 3.40 GHz.

5. Conclusions

- A modular Python computer code has been developed for the automatic optimization of prestressed concrete bridge decks using genetic algorithms. The Python code takes into consideration the construction stages, the redistribution of forces due to long-term effects (within Time Dependent Analysis) and all losses of post-tensioned tendon required by standards.
- The genetic optimization algorithm is based on finding the most efficient tendon geometry in a structure with the minimum prestressing force that satisfies the constraints given by design codes. The constraints are enforced by penalization with automatically scaled weights.
- A parametric study is provided for 3 common bridge deck cross-sections to demonstrate capabilities of the implementation. Eurocodes loads and combination factors, as well as Eurocodes functions for creep, shrinkage, relaxation and stress limitations are used for parametric study.
- Further development of the computer code for prestressed concrete bridge decks will focus on extending the optimization algorithms, including optimization of the cross-section dimensions, multiple tendons geometries, and variety of different optimization tools.

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