Understanding Felix Candela's 8-sided folded hypars

Comprendiendo los paraboloides hiperbólicos de 8 lados de Félix Candela.

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RESUMEN

Félix Candela (1910-1997) fue conocido por sus láminas de hormigón, como los paraboloides hiperbólicos (hypars). Creó sus «paraguas» invertidos de 4 lados combinando cuatro tímpanos de hypar y los de 8 lados introduciendo bisectrices en cada tímpano. Candela afirmaba que éstas aumentaban la rigidez global y la eficacia estructural con respecto a los de 4 lados, una afirmación nunca demostrada. Basándose en los hypars de 8 lados diseñados por Candela en Cuba y Ciudad de México, el estudio paramétrico realizado muestra cómo los paraguas de 8 lados presentan más deformaciones y tensiones que los de 4 lados, lo que refuta la hipótesis de Candela, aunque muestran una mayor resistencia al pandeo.

ABSTRACT

Félix Candela (1910–1997) was known for his thin concrete shells, such as hyperbolic paraboloids (hypars). He created his 4-sided inverted 'umbrellas' by arranging four tympans of a hypar surface, and his 8-sided forms (a 'folded hypar') by bisecting each tympan diagonally. Candela claimed these bisectors contributed to increase global stiffness and structural efficiency over 4-part umbrellas, a never-verified statement. Based on the two folded hypars designed by Candela in Cuba and Mexico City, the parametric study in this paper shows that 8-sided umbrellas exhibit larger deflections and stresses relative to 4-sided renditions, thus rebuking Candela's hypothesis, although they show increased resistance to shell buckling

PALABRAS CLAVE: Candela, Paraguas de 8 lados, Paraboloide hiperbólico, Método elementos finitos.
KEYWORDS: Candela, 8-sided umbrella, Hyperbolic paraboloid, Finite element modeling.

1. Introduction.

Félix Candela (1910–1997) was a Spanish-Mexican structural artist, most known for his thin shell concrete forms, especially for his hyperbolic paraboloids (hypars) [1] from which numerous forms were derived [2]. Traditionally fabricated from reinforced concrete, the geometric efficiency inherent to double curvature of hypars facilitates its ability to cover an area exceeding 150 m^2 while remaining only 40 mm thick. Another advantage, being a ruled surface, is the economy of construction as straight wooden elements can be adopted as formwork and straight steel bars can be used as reinforcement.

Candela arranged four quadrants of a hypar surface (tympans) in a manner leading to the formation of a 4-sided inverted umbrella. For a detailed architectural account of Candela's many works involving the hyperbolic paraboloid, the reader is referred to Garlock and Billington [3] and Faber [4]. To create the 8-sided form, each smooth tympan is bisected diagonally by a raised parabola, resulting in a folded hypar (Fig.1). Candela asserts the parabolic bisector contributes to the global stiffening of the shell, resulting in increased structural efficiency over its more modest 4-sided counterpart [4]. However, such a claim has never been independently verified via rigorous methods of structural analysis.

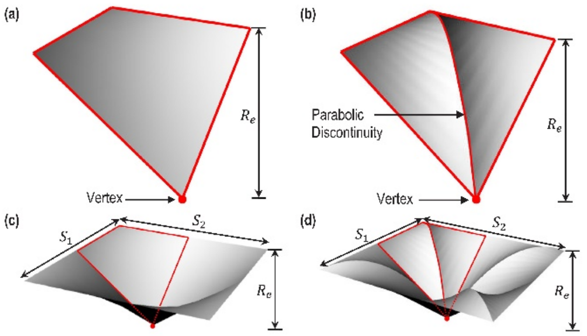

Figure 1. Tympan of (a) 4-sided and (b) 8-sided hypars used to form complete (c) 4-sided and (d) 8-sided umbrellas respectively.

This paper presents the findings of a parametric investigation [5] that has been carried out, based on the two folded hypars designed by Candela: the proposed Presidential Palace in Havana [6], Cuba (Fig. 2), and the Jamaica Wholesale Market (Fig. 3), built in Mexico City [7] between 1956 and 1957.

The approximate geometric dimensions of the Cuban umbrellas considered in this study are: Area=64m^2, *Re*=3m, *Rp*=3.31m and thickness=40mm, discerned from the architectural drawings.

The Jamaica Market contained two rows of twelve concrete umbrellas measuring approximately 18 m × 18 m in plan. The considered dimensions are Area=324 m^2, *Re*=3m, *Rp*=3.05 m and thickness = 40 mm.

Despite their massive area, the overall rise (*Re*) could not exceed 3 m between the vertex and roof edges per governmental decree. This resulted in a *Re*/A of 0.0092 m^{-1}, leading Candela to comment that "In this case, I'm afraid we passed the limit" [4] in likely reference to his ideal *Re*/A of no less than 0.015 m−1 to control excessive deflections. In response, Candela hoped to stiffen the hypars by incorporating a diagonal parabolic fold into each tympan, giving rise to the characteristic 8-sided form. After the Mexico City Earthquake in 1985, and despite the umbrellas themselves were structurally intact, it was eventually demolished due to the failure of supporting columns.

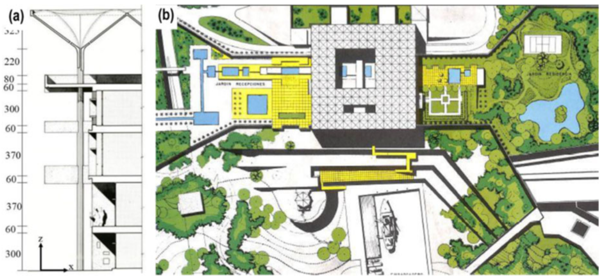

Figure 2. Façade elevation showing 8-sided umbrellas and (b) aerial view of the proposed Cuba Presidential Complex [5]

In particular, Candela's hypothesis regarding the enhanced performance of 8-sided umbrellas relative to 4-sided forms is examined via the quantification of structural deflections, flexural bending, and the flow of principal forces and stresses within the shell. A qualitative assessment into the buckling performance of 8-sided hypars is finally undertaken to evaluate Candela's belief in the superior resistance to buckling offered by thin-shelled umbrellas.

Figure 3. The Jamaica Market as captured after its completion (Photo: Department of Special Collections. Princeton University Library)

2. Geometric description

The 4-sided umbrella consists of four smooth tympans (Fig. 4) arranged such that the low points meet at the vertex coinciding with the column [3].

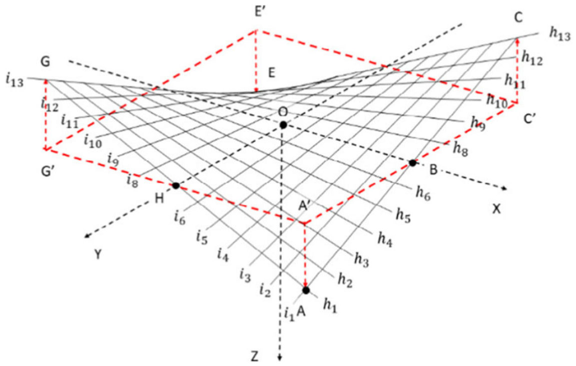

Figure 4. Construction of hypar surface via straight line generators (tympan bounded by straight edges)

8-sided umbrellas exhibit the same basic form as their 4-sided counterparts albeit with a parabolic discontinuity from each corner to the vertex as shown in Fig. 5. This discontinuity discretizes the otherwise smooth tympan into two sections bounded by two straight lines and a parabolic curve.

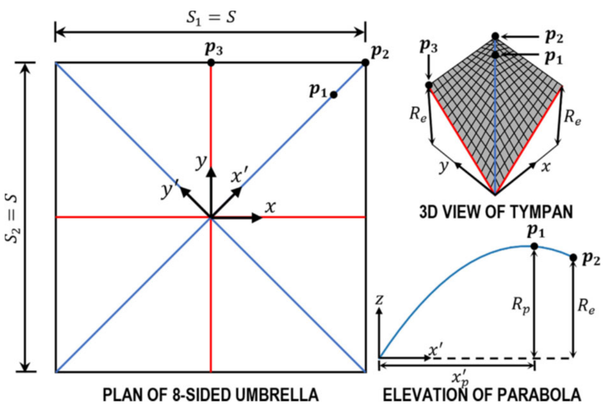

Figure 5. Geometric description of 8-sided umbrella

Note that the geometry of 8- sided umbrellas in this study are primary characterized by the following parameters:

- *A*= projected area of the square umbrella in plan view
- *Re* = vertical distance between the vertex and edges (edge rise)
- *Rp* =vertical distance between the vertex and apex of each parabola (parabola rise)

3. Case studies

Structural analyses were undertaken via a parametric study in quantifying the influence of geometry on the behavior.

Concrete properties constituting the Young's modulus, E, Poisson ratio, ν, and compressive strength, *fc'*, of the 40 mm thick shells were assumed to be 24.9 GPa, 0.2, and 27.6 MPa respectively. While the thickness of the shell was not explicitly stated, an assumption of 40 mm was adopted in this work on account of Candela's other umbrella designs.

Table 1. Parameters for the investigation based on Presidential Palace in Cuba. (*) Candela`s design

Type	Designation	R_e/A (m^{-1})	R_e (m)	R_p/R_e
4-sided	C4-47	0.047	3.00	N/A
	C4-30	0.030	1.92	
	C4-15	0.015	0.96	
	C4-10	0.0092	0.59	
	C4-5	0.005	0.32	
8-sided	**C8-47 (*)**	**0.047**	**3.00**	1.10
	C8-30	0.03	1.92	
	C8-15	0.015	0.96	
	C8-10	0.0092	0.59	
	C8-5	0.005	0.32	

Table 2. Parameters for the investigation based on Jamaica Market in Mexico. (*) Candela`s design

Type	Designation	R_e/A (m^{-1})	R_e (m)	R_p/R_e
4-sided	M4-15	0.015	4.86	N/A
	M4-10	0.0092	3.00	
	M4-5	0.005	1.62	
8-sided	M8-15	0.015	4.86	1.02
	M8-10 (*)	**0.0092**	3.00	
	M8-5	0.005	1.62	

The commercial finite element software SAP2000 [8] was utilized for the parametric study to which all hypars were modeled via thin shell elements. An optimization study was first conducted to determine the suitable mesh resolutions for both sets of models.

For purposes of comparative analyses, only the material self-weight was considered due to its significance on the structural response. Due to double symmetry of the umbrellas, only a single tympan quadrant was modeled, and the restraints of the boundary nodes were defined accordingly.

A total of 16 models were considered, as summarized in Tables 1 and 2 and shown in Fig. 6, there the true geometry of the Presidential Palace and Jamaica Market umbrellas are highlighted in bold and circled

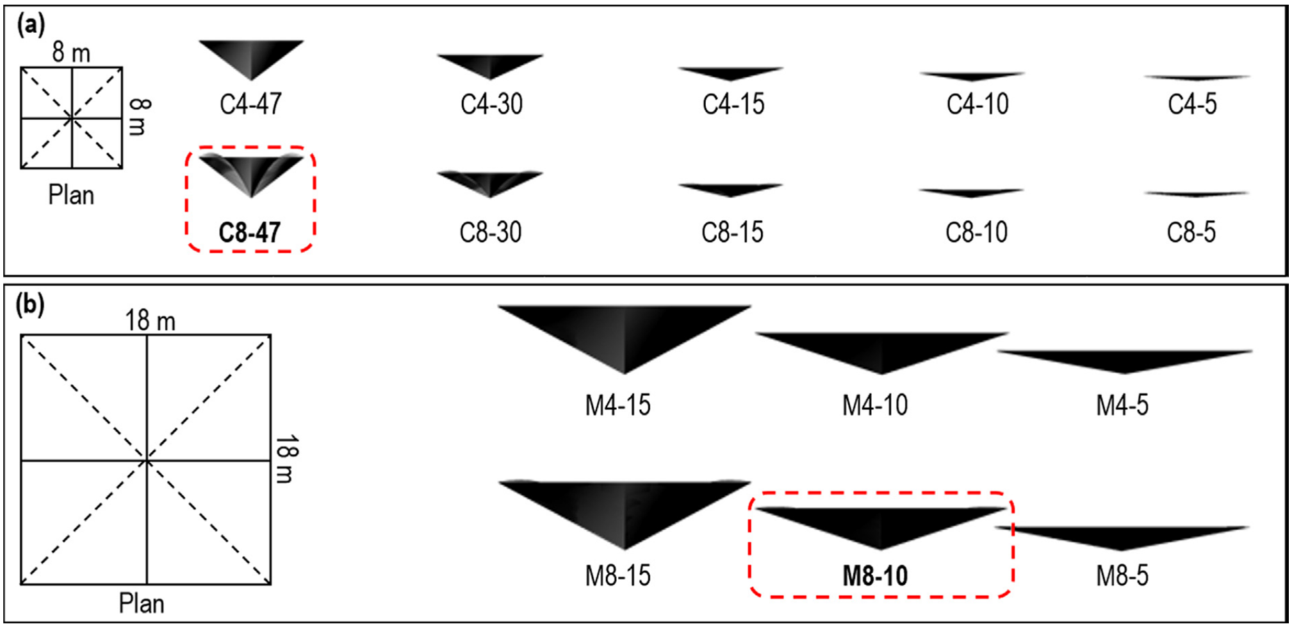

Figura 6. Scaled representation of 4 and 8-sided umbrellas derived from (a) the Cuba Presidential Palace and (b) Mexican Jamaica Market implemented for the parametric study. Dashed lines on plan indicate parabolic tympan bisectors for 8-sided umbrellas (C8 and M8). Red boxes encompass the true geometry of the two case studies.

4. Shell deflections

As expected, the largest deflections across all geometries under self-weight occurred at each of the four corners of the umbrella. Fig. 7 depicts the absolute corner deflection, δ, of each hypar configuration with respect to changes in Re/A for both the Cuba and Mexico shells. It is observed that increasing Re/A corresponds with a reduction in overall deflection.

In addition, all 8-sided umbrellas exhibited greater magnitudes of deflection relative to their 4-sided counterparts at any given Re/A. This behavior is in direct contrast to Candela's intention of providing additional stiffness to traditional 4-sided hypars via the bisection of each tympan with a parabolic discontinuity.

The corner deflections were subsequently normalized by the valley length of each tympan, L, for each geometric configuration where

$$L = \sqrt{R_e^2 + \left(\frac{S}{2}\right)^2} \qquad \textbf{(1)}$$

which enables the determination of L/δ ratios as shown in Fig. 8, and allows for the direct comparison between deflection limits stipulated within ACI 318-14 [9], requiring ratios no less than 180 or 240 when neglecting or including the support of non-structural elements respectively

It can be observed that all geometries exhibiting $Re/A = 0.005\ \mathrm{m}^{-1}$ would fail to satisfy this conservative critical threshold of $L/\delta \geq 240$. In addition, while the 8-sided Presidential Palace umbrellas in Cuba would have proven satisfactory from a serviceability perspective, the

Mexican Jamaica Market hypars with L/δ =116 falls well short of ACI 318 stipulations. Ironically, if Candela had not incorporated parabolic folds into the Jamaica Market as a misguided attempt to limit deflections, its 4-sided equivalent with L/δ = 246 would have achieved compliance with modern serviceability standards.

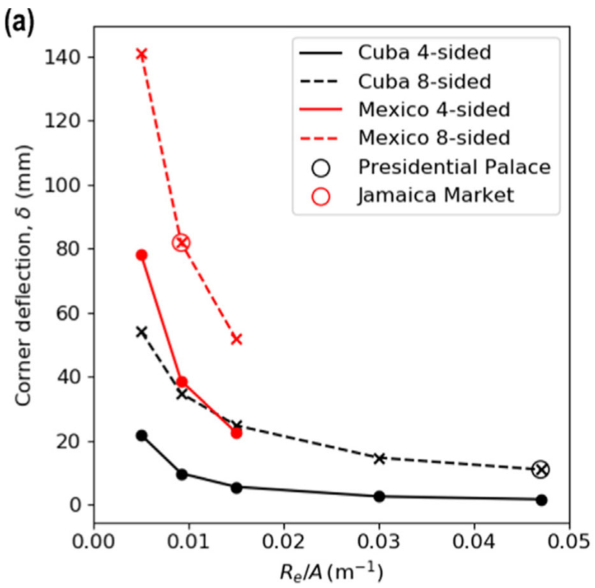

Figura 7. Absolute corner deflections of 4 and 8-sided umbrellas, function of *Re/A*

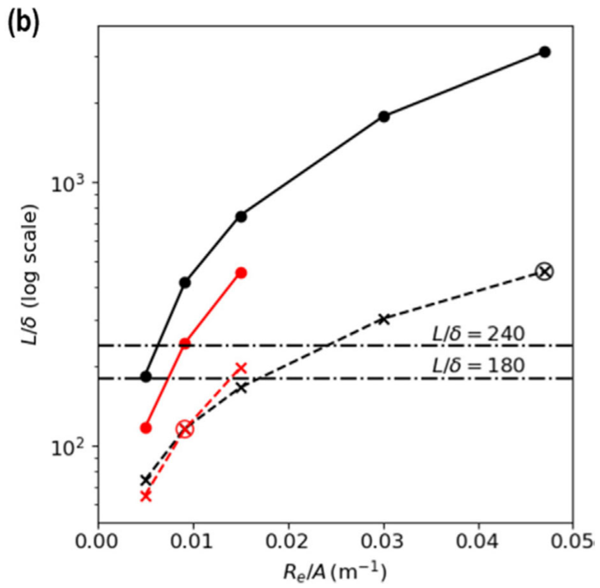

Figura 8. Deflections normalized by valley length of 4 and 8-sided umbrellas, function of *Re/A*.

5. Flexural action and principal stresses

Fig. 9 illustrates the distribution of bending moment along the local x and y axes for 4 (C4-47) and 8-sided (C8-47) renditions of the Cuba Presidential Palace as representative of the overall hypar response.

Fig. 10 presents a qualitative overview of the distribution and orientation of tensile and compressive principal forces (acting in-plane about the neutral axis of the shell) for 4 and 8-sided umbrellas pertaining to the Cuba Presidential Palace exhibiting Re/A ratios of 0.047 m^{-1} (Fig. 10a and b) and 0.005 m−1 (Fig. 10c and d).

It is therefore evident that this raised parabola introduces a sharp discontinuity which disrupts the flow of forces through the otherwise smooth tympan.

The maximum bending moment (in absolute terms) and maximum tensile/compressive principal stresses are subsequently summarized for all hypar geometries constituting the parametric study via Fig. 11 and 12. Ultimately, it is clear that 8-sided hypars generally exhibit greater moments and principal stresses than their simpler 4-sided renditions for a given Re/A. The principal stresses of such hypars under self-weight were further compared against the concrete capacity. When adopting a compressive strength *fc*' of 27.6 MPa (4000 psi), the corresponding tensile capacity was estimated via $ft \approx 0.62\,(fc')^{1/2}$ = 3.3 MPa. It is witnessed that while all geometries failed to exceed their compression capacity, only C4-15, C4-30, and C4-47 were within the tensile limit for unreinforced concrete.

Moreover, the location of maximum tension for the shallower 8-sided hypars C8-5, C8-10, and C8-15 were along the free edges. Steel reinforcement must therefore be incorporated along all edges and the parabolic discontinuity when considering the design of 8-sided hypar umbrellas.

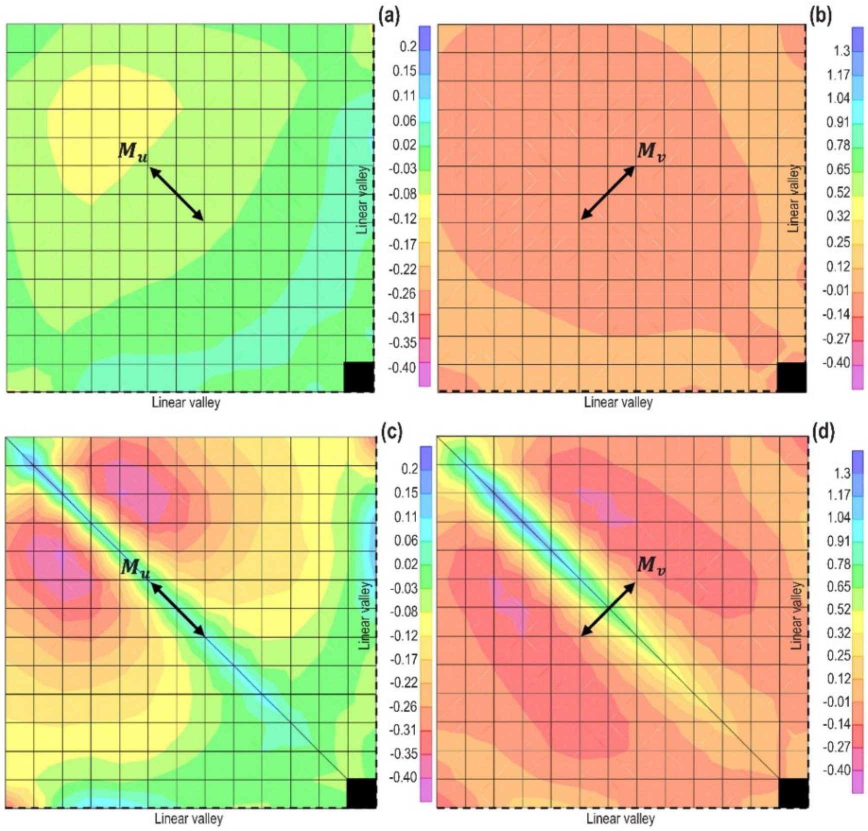

Figure 9. Bending moment distribution of C4-47 along local (a) u and (b) v axis, and C8-47 along local (c) u and (d) v axis. All units in kNm/m

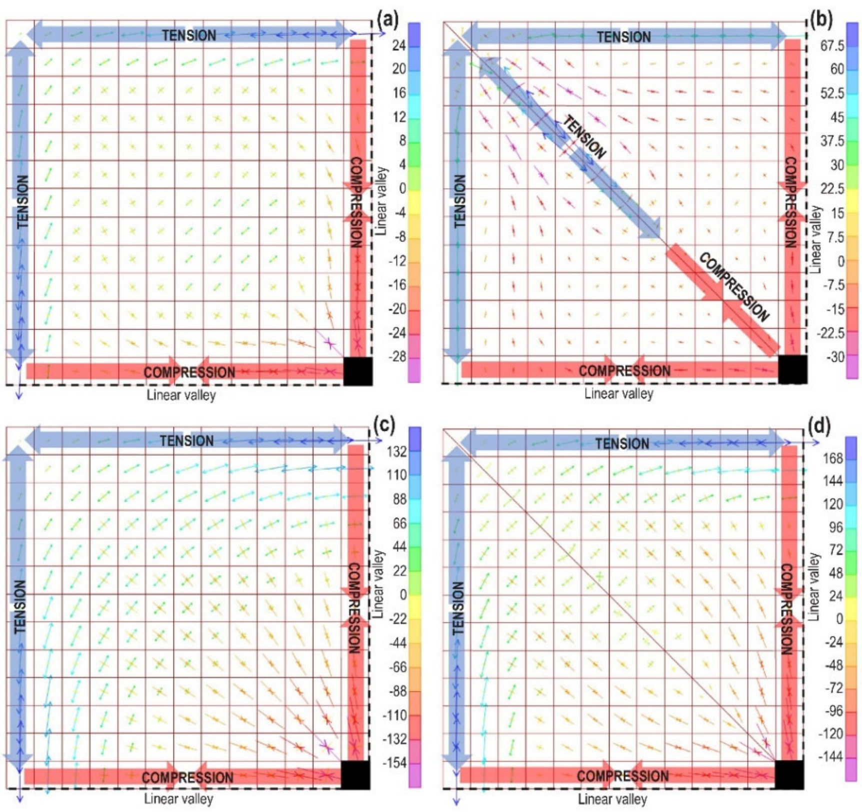

Figure 10. Distribution and orientation of principal forces for (a) C4-47, (b) C8-47, (c) C4-5, and (d) C8-5 showing primary regions of tension and compression. All units in kN/m.

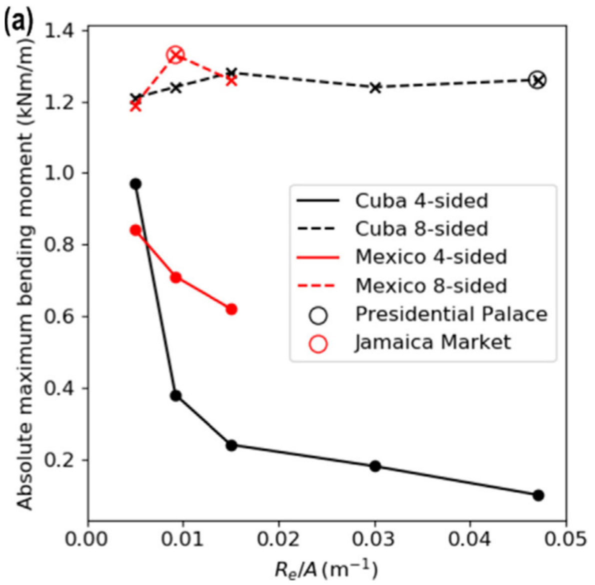

Figura 11. Maximum absolute bending moment for *Re*/A ratios pertaining to the parametric study.

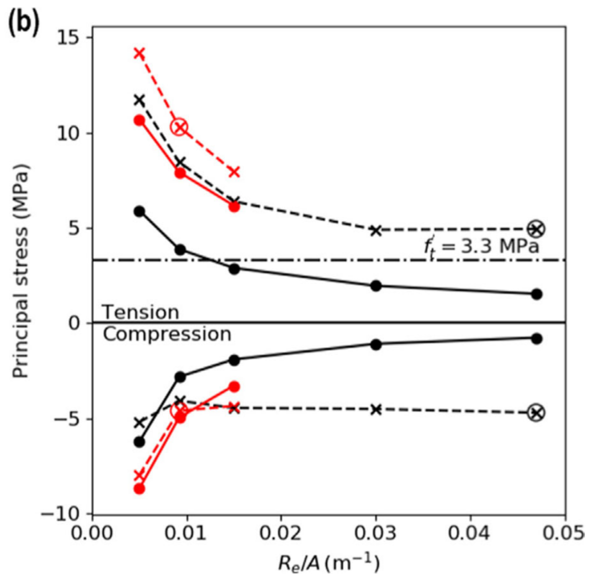

Figura 12. Tensile/compressive principal stresses for *Re*/A ratios pertaining to the parametric study.

6. Influence of parabolic rise

The parametric investigation has thus far focused only on the influence of edge rise (*Re*) while maintaining a constant *Rp*/*Re* ratio.

A hypar with each side measuring 8 m (giving a projected area of 64 m^2) was considered, reflecting the footprint of the Cuban umbrellas. A total of three edge rises were examined consisting of 0.96 m, 1.92 m, and 3 m, which correspond to *Re*/A ratios of 0.015 m^{-1}, 0.03 m^{-1}, and 0.047 m^{-1} respectively. For each edge rise, the *Rp*/*Re* ratio was varied from unity (reflecting a conventional 4-sided hypar) to 1.5 across four uniform increments of 0.125 as illustrated per Fig. 13a. Fig. 13b and c respectively denote the corner deflection and maximum bending moment present within each tympan as a function of *Rp*/*Re* for each *Re*/*A*.

It is seen that for any given *Rp*/*Re*, increasing *Re*/*A* coincides with a reduction in the corner deflection and moment demand due to the increased stiffness of the shell. In addition, the location of maximum moment for the 8-sided hypars all exist at the parabolic apex consistent with Fig. 9d.

For *Re*/*A* of 0.003, 0.047 and 0.015 m−1, both the deflection and bending appear to increase up to approximately *Rp*/*Re* = 1.25, 1.25 and 1.375, respectively, before decreasing as the apex is further risen. These results show how, while the discontinuity disrupts the flow of forces leading to the presence of stress concentrations, the sheer steepness of the parabola contributes to the global stiffness of the tympan to a certain extent at larger *Rp*/*Re* ratios.

7. Shell buckling

A simplified qualitative assessment of buckling is therefore explored for the Cuba Presidential Palace (C8-47) and Jamaica Market (M8-10) umbrellas. As such, only the linear valleys were examined due to the presence of maximum compression occurring in these regions.

A representative effective width, *w*, was subsequently selected upon inspection of the finite element model (Fig. 14). This width was adopted for determination of the out-of-plane area moment of inertia, *I*, for use within Euler's buckling equation (2)

$$P_{cr} = \frac{\pi^2 EI}{(k_b \cdot L_b)^2} \tag{2}$$

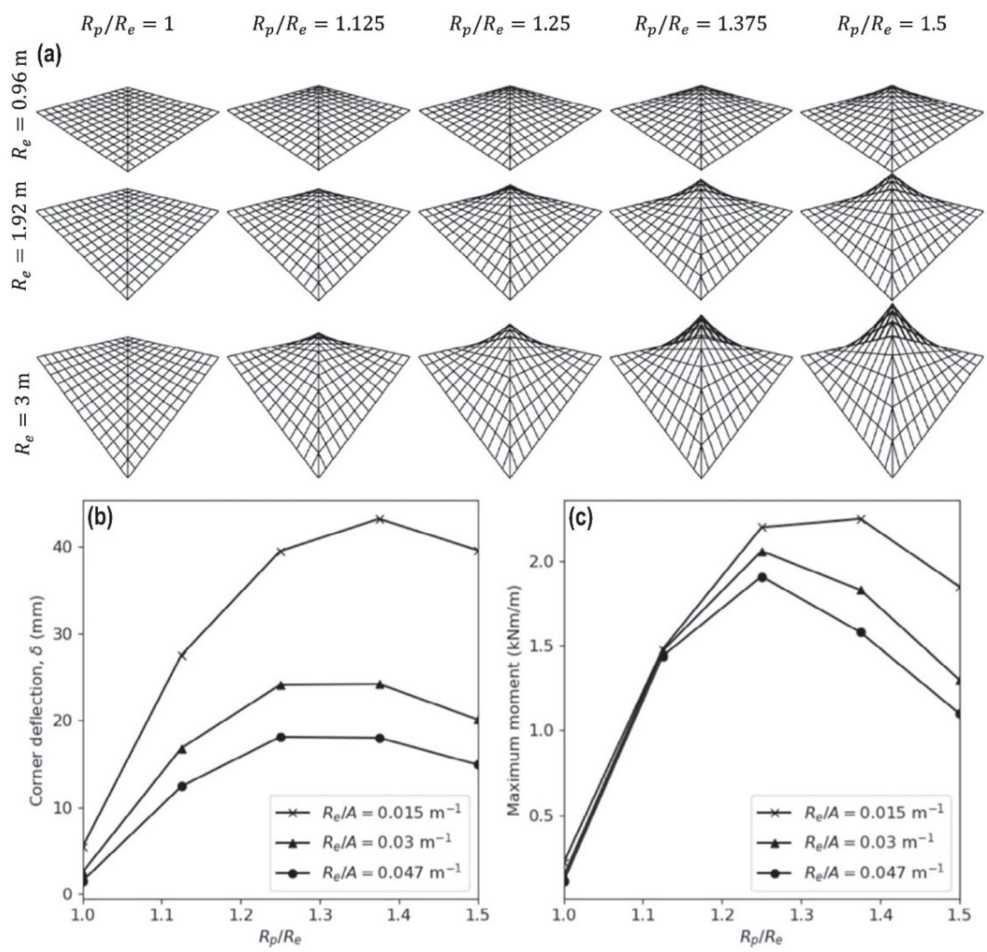

Figura 13. (a) Visualization of hypar geometries adopted for the parametric study and results pertaining to the (b) corner deflection and (c) maximum bending moment as a function of *Rp/ Re* and *Re/ A*.

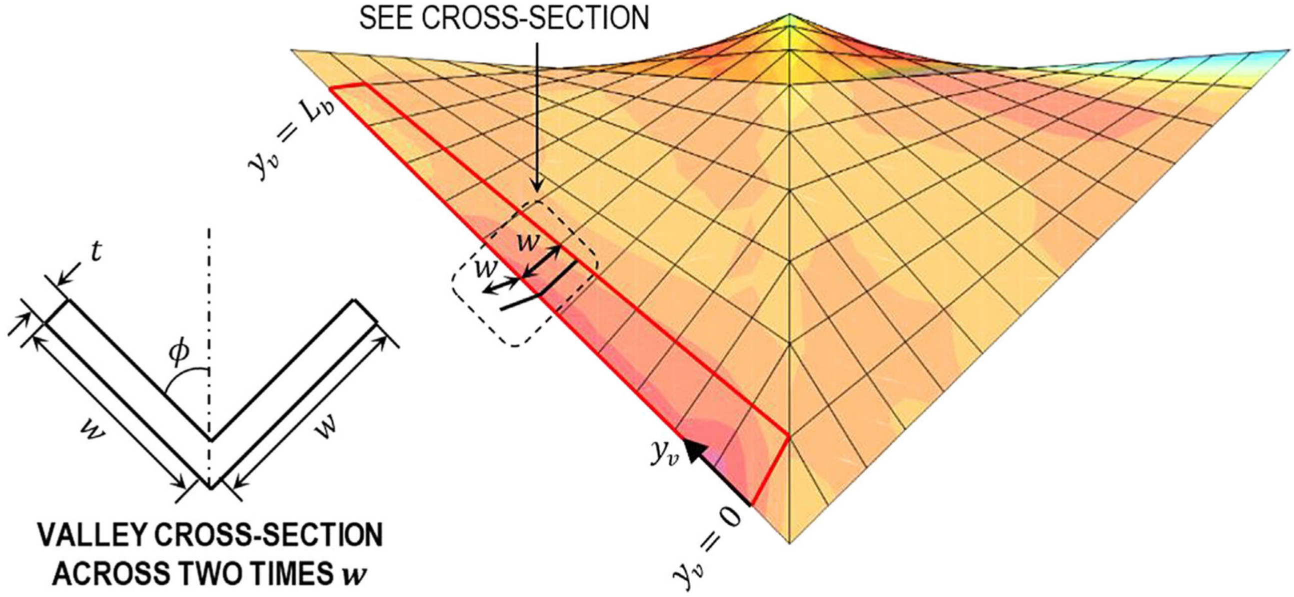

Figura 14. Illustration of simplified technique to estimate the buckling performance of 4 and 8-sided umbrella

where P_{cr} is the critical buckling force and L_b denotes the buckling length. Note that L_b does not correspond to the entire valley length, L, since the final 0.4 m and 0.5 m of the Cuba and

Mexico umbrella valleys, respectively, are in tension. An effective length factor, *ke*, of unity is therefore adopted, reflecting a column pinned at the edge and vertex. The buckling length for C8-47 and M8-10 were respectively determined to be 4.6 m and 9 m, with corresponding effective widths of 0.3 m and 0.9 m.

The buckling performance of the 8-sided Presidential Palace and Jamaica Market umbrellas was also compared against their 4-sided renditions. From Table 3, it is seen that 8-sided forms exhibit both larger *Pcr* and smaller compressive demands, *P*, over the effective width along the valleys, resulting in an improved resistance to buckling relative to 4-sided hypars.

Table 3. Buckling analysis for 4 and 8-sided renditions of Presidential Palace(a) and Jamaica Market Umbrellas(b).

Umbrella	w (m)	L_b (m)	Pcr (kN)	P (kN)	P/Pcr
C4-47	0.3	4.6	207	6.95	0.034
C4-47(a)			529	6.05	0.11
M4-10	0.9	9.0	389	103	0.265
M8-10(b)			614	83.2	0.136

Conclusions

As main results, 8-sided umbrellas exhibit larger deflections and stresses relative to 4-sided renditions, thus rebuking Candela's hypothesis concerning the improvement to structural efficiency offered by the parabolic fold.

While corner deflections and principal stresses generally decrease with increasing curvature, the discontinuity present in 8-sided forms disrupt the flow of internal forces, resulting in stress concentrations at the parabolic apex manifesting as large moment demands. However, it was demonstrated that 8-sided hypars exhibit increased resistance to shell buckling over 4-sided variants as revealed through a simplified analytical approach.

Acknowledgments

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