

# Dynamic response of cable-stayed footbridges with tuned mass dampers through stochastic analysis

*Respuesta dinámica de puentes peatonales atirantados con amortiguadores de masa sintonizada a través de análisis estocásticos*

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## RESUMEN

Este artículo muestra una evaluación exhaustiva de la respuesta dinámica sin y con amortiguadores de masa sintonizada en un puente peatonal atirantado de 60 m de longitud bajo diferentes densidades de peatones y actividades en dirección vertical y lateral, considerando un modelo de carga peatonal estocástico. La influencia de varios parámetros, como la relación de masa, la longitud del vano, las densidades de peatones, la forma de caminar de los peatones, la forma de la torre, y la ubicación de los amortiguadores de masa sintonizada, se mostrarán con la finalidad de establecer un conjunto de recomendaciones que puedan considerarse en futuros diseños.

## ABSTRACT

This paper shows an exhaustive evaluation of the dynamic response with and without tuned mass dampers in a cable-stayed footbridge with a span of 60 m under different pedestrian densities and activities in vertical and lateral direction, considering a stochastic pedestrian flow type. The influence of several parameters such as mass ratio, span length, pedestrian densities, pedestrian activities, pylon shape, and location of the tuned mass dampers, will be shown in order to set up a set of recommendations that can be considered in future designs.

**PALABRAS CLAVE:** Tuned mass dampers, cable-stayed footbridges, human-induced vibration, dynamic response

## 1. Introduction

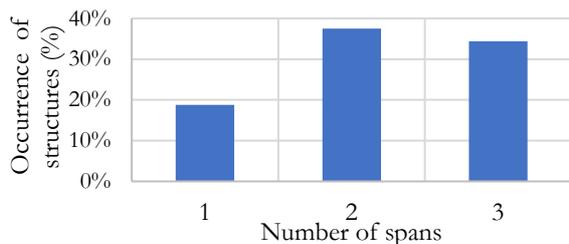
Cable-stayed footbridges are being built lighter and slenderer due to the recent developments in the design methodology, construction techniques and material types. However, these improvements reduce the mass and the damping ratio, increasing the human-induced vibration (serviceability problems). Some cable-stayed footbridges which have experienced unexpected vibrations are the Forchheim footbridge in Germany [1], Toda cable-stayed footbridge Park

in Japan [2], The Footbridge Skoda in Poland [3], and the Pilsen Footbridge in Czech Republic [4]. In these cases, supplemental damping devices were employed to satisfy the serviceability criteria. Gaining a better understanding of the structural response of footbridges with and without tuned mass dampers is necessary to identify the cases that can have serviceability problems and propose design recommendations. Therefore, the main objective of this paper is to analyse the structural response of a cable-stayed

footbridge with a span length of 50 m under a stochastic pedestrian load model [5,6,7] with one tower (see sketch in Table 1) which was selected based on data published about conventional cable-stayed bridges that have already constructed [e.g., 8,9,10 and past footbridges conferences]. This benchmark case was studied by Ramos-Moreno [5] and it was showed that the serviceability limit state (SLS) of vibration is not satisfied without supplemental damping devices. Thus, the geometry and modal analysis of the cable-stayed footbridge are described. Then the dynamic response with and without tuned mass dampers in the vertical and the lateral direction is shown. The tuned mass damper properties to fulfil the maximum comfort criteria in both directions and also to avoid the “lock-in” effect in the lateral direction is highlighted. In this work, the TMDs characteristics (optimal damping frequency and damping ratio) were obtained according to the Asami criteria [11].

## 2. Characteristics of the cable-stayed footbridge

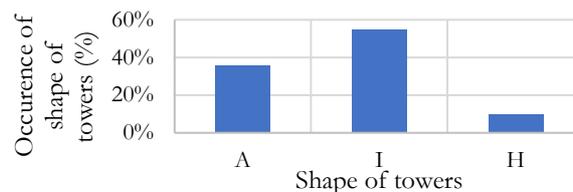
A cable-stayed footbridge is considered to be an efficient and economical structure, with stay cables supporting the deck from one or more pylons or towers. For the evaluation of the dynamic response, a footbridge with a very common cross-section (a composite section made of two steel girders and a reinforced concrete slab) is analysed [7]. According to data found in literature, the most common cable-stayed footbridges have two and three spans (Figure 1). Their average main span lengths are approximately 50 and 100 m for two and three spans, respectively. The side to main span ratio varies from 0.20 to 0.45 [8].



**Figure 1: Occurrence of cable-stayed footbridges according to the number of spans based in a dataset of this typology**

The pylon is frequently made of either structural concrete or steel. The pylon usually has either an I- (single pylon), A-, inverted Y-, or diamond shape. The most common pylon shapes found in the literature review are the I- and A-shaped, and the less used is the H-shaped pylon (Figure 2). In an A-shaped pylon, the foundations are connected to avoid transmitting a large horizontal action to the ground.

For this research the performance of I-, A-, and H- shaped pylons will be evaluated in order to analyse the impact that can have in the dynamic response. The main geometric characteristics of the benchmark case can be seen in Table 1 [7]. The material properties of the cable-stayed footbridges are selected according to guidelines and standards (Table 2). The material damping ratio is selected according to the structural material (0.4 %) [11].



**Figure 2: Occurrence of cable-stayed footbridges according to the pylon shape**

Main span length (L)	50 (m)
Side span length ( $L_k$ )	0.20 L (m)
Deck depth	L/100 (m)
Deck width	4 m
Cable-system arrangement	Semi-fan (with 0.5 m spacing between anchorages of stay cables at the top of the pylon)
Pylon shape	I, H and A

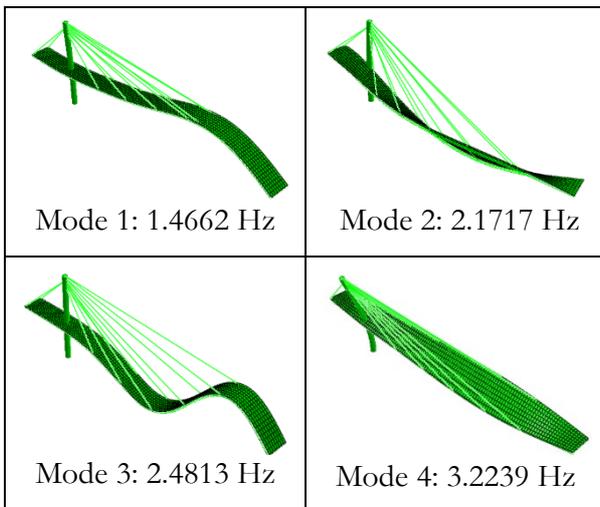
**Table 1: Geometric characteristics of the cable-stayed footbridges**

Structural concrete		Structural steel	
Compressive strength	$f_{ck} = 40 \text{ MPa}$	Yield strength	$f_{s,y} = 355 \text{ MPa}$
Density	$2500 \text{ kg/m}^3$	Density	$7850 \text{ kg/m}^3$
Poisson's ratio	0.2	Poisson's ratio	0.3
Young's modulus	35 GPa	Young's modulus	210 GPa

**Table 2: Characteristics of the structural materials used in the deck and the pylons for the deck and pylons for cable-stayed footbridges in this work**

### 3. Results

Cable-stayed footbridges are characterised to have lower natural frequencies that can lie or match with the range of the pedestrian frequencies, amplifying the dynamic response. In this work, a modal analysis was performed to check if the natural frequencies lie near the step frequencies of the pedestrians. Figure 3 shows how multiple vertical modes have frequencies that are within the potential range of pedestrian frequencies [13].



**Figure 3: Modal frequencies of cable-stayed footbridges with a span length of 50 m with an I-shaped pylon**

The dynamic response is checked according to the comfort levels given in codes, standards, and guidelines to analyse if the serviceability criteria is fulfilled. For the cases where the serviceability criteria is not fulfilled, tuned mass dampers will be employed to mitigate human-induced vibrations. In addition, the impact and the

efficiency of these devices in reducing the dynamic response will be assessed. The TMD efficiency can be measured as the percentage of the peak accelerations, prior to the implementation of the TMD, that would be cancelled after its installation. The following sections show the accelerations of the cable-stayed footbridges with and without tuned mass dampers, and its comparison to the different comfort levels (maximum, minimum and mean) [14,15].

#### *3.1 Accelerations without tuned mass dampers for cable-stayed footbridges with an I-shaped pylon*

Figure 4 shows the vertical and lateral peak accelerations, for commuters (going to or coming back from work) commuting and for pedestrians engaged in leisure activities, in a cable-stayed footbridge with a span length of 50 m. The comfort level in the vertical direction corresponds to a minimum comfort for commuting and leisure activities, for the range of the pedestrian densities, except for leisure activities with a pedestrian density of  $0.2 \text{ ped/m}^2$  which has mean comfort level. In the lateral direction when the pedestrian density is  $1.0 \text{ ped/m}^2$ , the comfort is minimum. Otherwise, the lateral acceleration provides mean and maximum comfort levels. However, in most of these cases there is a potential risk of the pedestrians locking-in or synchronizing with the vibrations of the structure in the lateral direction. This effect, that needs to be checked, could happen when the

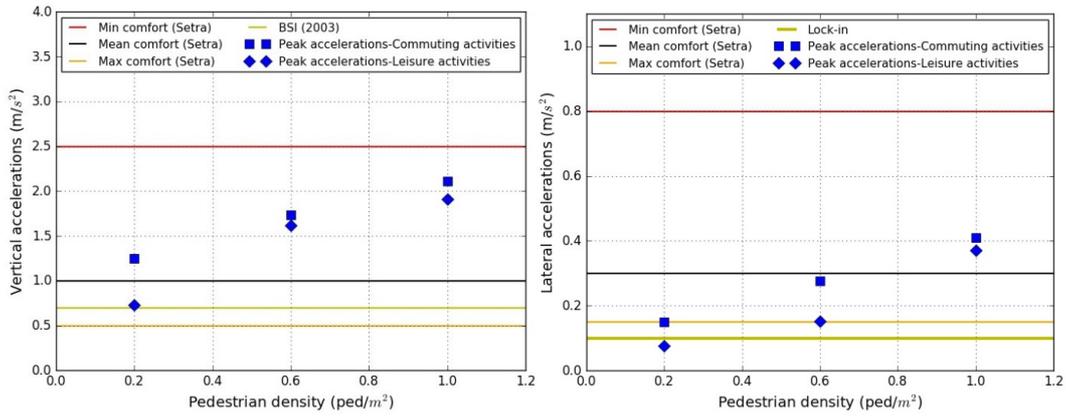


Figure 4: Maximum vertical and lateral peak accelerations for cable-stayed footbridges with a span length of 50 m under different pedestrian densities for commuting and leisure activities

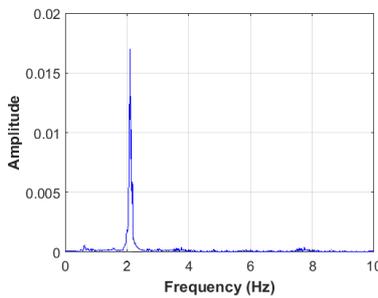


Figure 5: Frequency domain analysis at 25 m ( $L/2$ ) from the pylon for a CSF of a span length of 50 m under a pedestrian density of 0.6  $\text{ped}/\text{m}^2$  for commuting activities

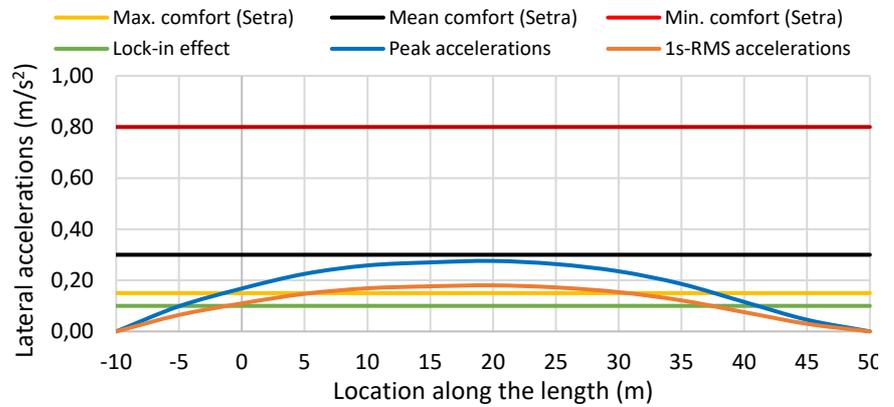


Figure 6: Peak and 1s-RMS lateral accelerations for a CSF of a span length of 50 m under a pedestrian density of 0.6  $\text{ped}/\text{m}^2$  for commuting activities

lateral accelerations are beyond  $0.1 \text{ m}/\text{s}^2$  (see Figure 4).

According to the analysis in the frequency domain in the lateral direction (Figure 5), the natural frequency of the critical mode of the footbridge is 2.17 Hz, which corresponds to the first lateral mode (see mode 2 in Figure 3). Figure 6 shows the lateral peak and root-mean-squared (rms) accelerations along the deck, which are within the mean comfort level. However, in this case, the lateral accelerations are larger than the  $0.1 \text{ m}/\text{s}^2$  limit, and pedestrians are likely to lock-in with the lateral vibration of the footbridge, inducing in turn larger lateral acceleration. Therefore, TMDs are implemented to mitigate

the lateral accelerations and eliminate this synchronization effect.

### 3.2 Accelerations with tuned mass dampers for cable-stayed footbridges with an I-shaped pylon

After carrying out an initial dynamic analysis, showed in the previous section, several cases where the accelerations exceeded the vibration limits were identified. As a result, on further analysis the impact of employing tuned mass dampers to mitigate the human-induced vibrations is investigated. Following a systematic procedure to include tuned mass dampers [6], the structural natural frequencies and the modal shapes were obtained, with the aim to identify

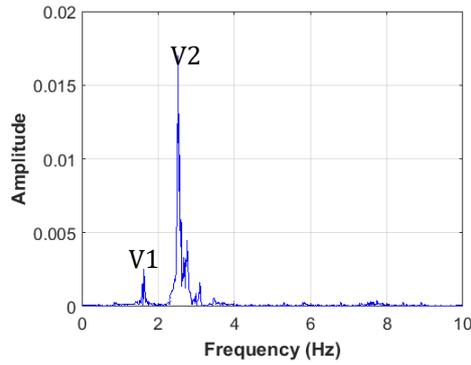


Figure 7 Frequency domain analysis at 18.5 m from the pylon in the vertical direction for a cable-stayed footbridge with a span length of 50 m under a pedestrian density of 0.6 ped/m<sup>2</sup> for commuting activities

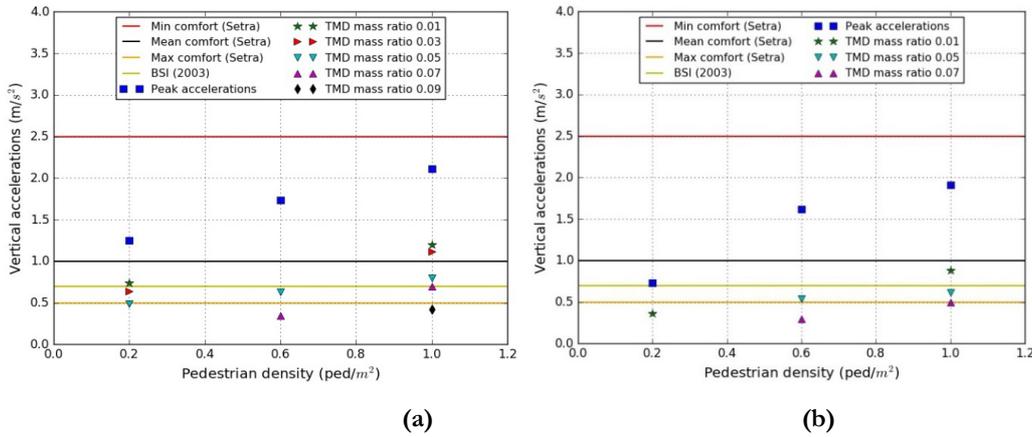
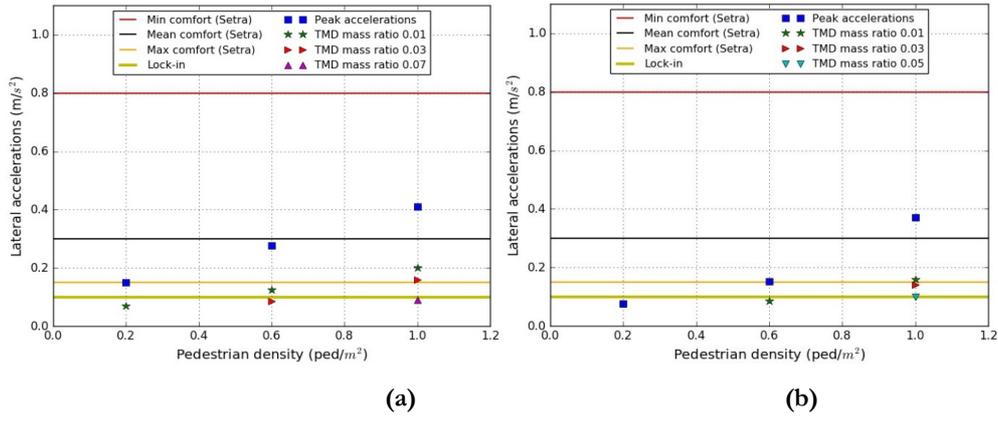


Figure 8 Peak vertical accelerations for cable-stayed footbridges with a span length of 50 m under a pedestrian density of 0.2, 0.6 and 1.0 ped/m<sup>2</sup> when different TMD mass ratios are employed for (a) commuting and (b) leisure activities

potential critical modes which can be in the range of pedestrian vertical and lateral frequencies. In addition, the frequency domain of the accelerations to be mitigated was obtained, in order to identify the frequencies of the modes with larger contributions in the response. Figure 7 shows the largest contribution made by the second mode in vertical direction (2.48 Hz), which lies in the range of pedestrian's frequencies. In this case, the maximum vertical acceleration does not fulfil the serviceability criteria. Therefore, a TMD is employed at the location where the vibrational mode linked to this vibrational frequency has the larger modal coordinate.

Figure 8 shows the required mass ratio of the tuned mass damper according to the different

comfort levels and for different design scenarios (pedestrian densities). The maximum comfort is achieved when the accelerations are not perceptible by pedestrians while they are crossing the bridge, mean comfort refers to situations when the accelerations are merely perceptible by pedestrians and the minimum comfort is reached when the accelerations are perceptible by pedestrians. Beyond the minimum comfort the accelerations are intolerable [15]. The lateral accelerations are limited to avoid lock-in effect as Setra guideline highlighted [15]. This effect is caused when pedestrians synchronize their step frequencies with the frequency associated with the lateral vibration of the structure. By implementing a TMD, the accelerations were significantly reduced with an average TMD efficiency of 75%.



**Figure 9** Peak lateral accelerations for CSFs with a span length of 50 m under a pedestrian density of 0.2, 0.6 and 1.0 ped/m<sup>2</sup> when different TMD mass ratios are employed for (a) commuting and (b) leisure activities

The mean comfort for commuting activities in the vertical direction can be achieved with a TMD mass ratio ranging from 0.01 to 0.07, depending on the pedestrian densities. If the maximum comfort in commuting activities is required, the necessary TMD mass ratio varies from 0.05 to 0.09. For leisure activities the mass ratio to obtain the mean comfort varies from 0.01 to 0.05 and to obtain the maximum comfort it varies from 0.01 to 0.07 depending on the pedestrian densities. To avoid the lock-in effect for commuting activities, the range for the TMD mass ratio varies from 0.01 to 0.07 depending on the pedestrian densities (Figure 9 (a)). For a pedestrian density of 0.2 ped/m<sup>2</sup> for leisure activities the serviceability criteria and the “lock-in” effect is fulfilled without the necessity of supplemental damping devices. A tuned mass damper with a mass ratio ranging from 0.01 to 0.05 is required to avoid the lock-in effect for CSF under pedestrian densities of 0.6 and 1.0 ped/m<sup>2</sup> (Figure 9 (b)).

### 3.3 Parametric analysis with H-, and A-shaped pylons

This section shows a parametric study of the dynamic response in vertical and lateral direction for cable-stayed footbridges with different pylon shapes. These pylon shapes are very common, as a consequence of structural and aesthetical consideration. The relevance of its shape in the dynamic response is analysed herein.

Figure 10 shows the analysis of the vertical accelerations with different pylon shapes. It is worth noting that the accelerations for the I- and the A-shaped pylons are very similar with differences smaller than 12%, leading to minimum comfort levels. The H-shaped pylon lead to larger vertical accelerations, with the minimum comfort being exceeded for pedestrian densities of 1.0 ped/m<sup>2</sup>. These results show a clear tendency in terms of performance, with the I-shaped pylons better than the A-shaped pylons, and these in turn better than the H-shaped pylons. Nevertheless, the difference in the accelerations are not large enough to have implications in terms of comfort levels. Moreover, the same tuned mass dampers that were employed for an I-shaped pylon (Figure 8) can be used herein for a cable-stayed footbridge with an A-shaped pylon to fulfil mean and maximum comfort. However, these dampers are not enough to minimize the dynamic response when an H-shaped pylon is employed. To fulfil the mean and maximum comfort using an A-shaped pylon to employ TMD mass ratios ranging from 0.01 to 0.09, and from 0.01 to 0.07 are required for commuting and leisure activities, respectively.

Lateral accelerations analyses are carried out in cable-stayed footbridges with different pylon shapes (Figure 11). The maximum lateral accelerations in the deck when I- or A-shaped pylons are used are very similar, as previously observed for vertical accelerations.

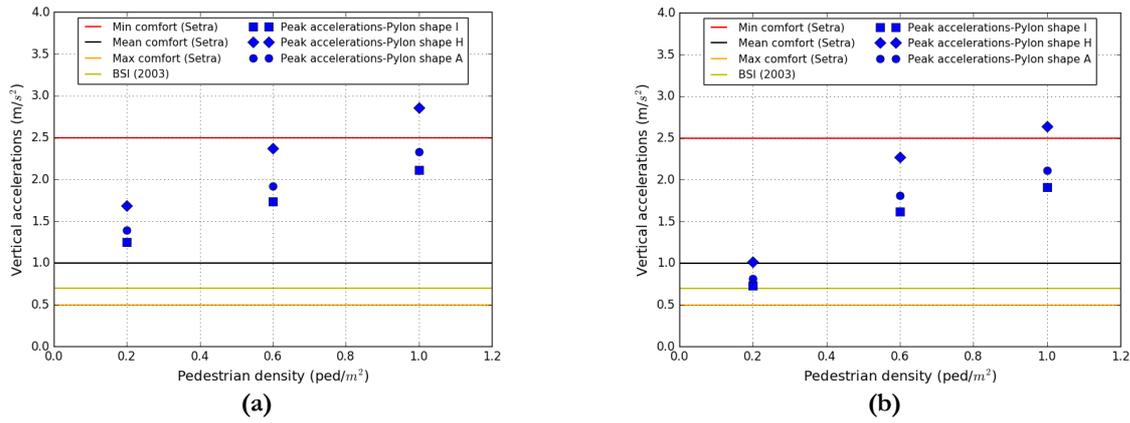


Figure 10: Peak vertical accelerations for CSFs with a span length of 50 m under different pedestrian densities for (a) commuting and (b) leisure activities for different pylon shapes

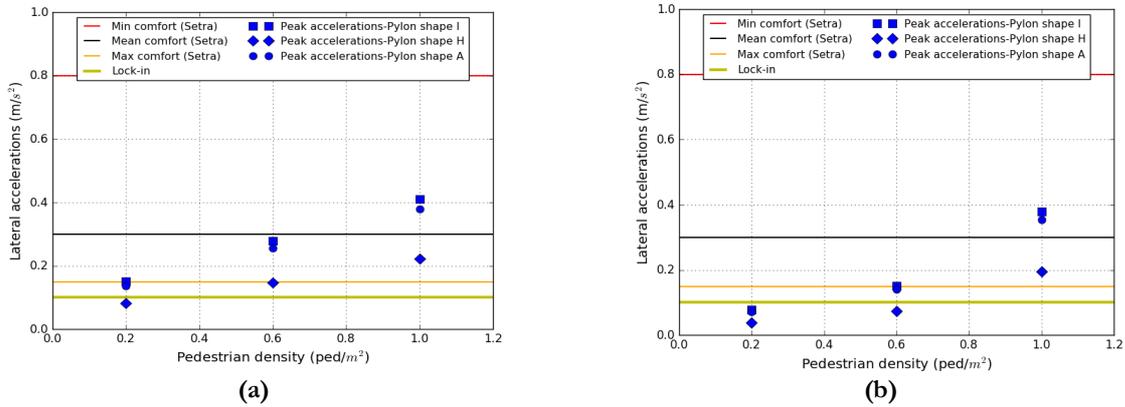


Figure 11 Peak lateral accelerations for CSFs with a span length of 50 m under different pedestrian densities for (a) commuting and (b) leisure activities for different pylon shapes

Nevertheless, the H- shaped pylons lead to the smallest lateral accelerations (with reductions in the lateral accelerations of up to 47% in comparison to the A- and I- shaped pylon), contrary to what was observed for vertical accelerations. In the cases when a pylon shape “I” and “A” were employed, the lateral accelerations are similar. These results show that the shape of the pylon also has an impact in the lateral accelerations.

#### 4. Discussion and conclusions

The main purpose of this paper was to evaluate the dynamic response of a cable-stayed footbridge with a span length of 50 m with one tower with and without tuned mass dampers considering different pylon shapes (A, H and I), pedestrian densities (0.2, 0.6, 1.0 ped/m<sup>2</sup>) and activities (commuting and leisure). Initially, the frequency domain analyses of these cable-stayed footbridges show that the frequencies of the

structure lies near the pedestrian step frequencies in vertical and lateral direction. Therefore, the dynamic response is enhanced due to resonant effect, so the serviceability criteria is not fulfilled. The analyses of the performance of cable-stayed footbridges with and without TMDs considering the different parameters allows the assessment of these devices in terms of fulfilling the serviceability criteria. In the vertical and lateral direction, results demonstrate that the tuned mass dampers significantly reduced the accelerations with TMD efficiencies up to 88%. Therefore, designers should consider the option of employing tuned mass damper devices in footbridges under pedestrian loading at the design stage when the accelerations exceed the comfort limits. It is worth emphasised that the tuned mass damper should be installed at the location of the maximum amplitude of the vibrational vibration mode which has a larger

contribution in the response. This mode can be identified through a frequency domain analysis.

A-shaped pylons are those which have a better performance when both vertical and lateral accelerations are considered. I-shaped pylons are the best ones when considering vertical accelerations, but have the worst performance of those assessed when considering the lateral accelerations. Contrary, H-shaped pylons are the best ones when considering lateral accelerations, but have the worst performance of those assessed when considering the vertical accelerations.

The dynamic analysis carried out in this cable-stayed footbridge with tuned mass dampers revealed a significant reduction in the accelerations when these devices are employed in the vertical and lateral direction and hence can be used as a part of the design stage (not only after construction) to avoid serviceability problems.

## 5. Acknowledgements

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