Dynamic amplification factors of cable-stayed footbridges*

Factores de amplificación dinámica para puentes peatonales atirantados

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RESUMEN

El artículo compara la respuesta estática y dinámica de un puente atirantado peatonal obtenida en simulaciones numéricas bajo una serie de situaciones en servicio. Las situaciones de servicio se han representado con un modelo de cargas novedoso. De la comparación de resultados se concluye que (1) las deformaciones estáticas del puente no pueden usarse para inferir las aceleraciones verticales máximas en servicio y (2) ciertos eventos con densidades de peatones en condiciones de servicio pueden generar tensiones en los elementos estructurales mayores que las de la sobrecarga estática de diseño por lo que hay que diseñar en ELU con resultados dinámicos originados por densidades de peatones en servicio.

ABSTRACT

The paper compares the static and dynamic performance of a cable-stayed footbridge under the effect of different serviceability pedestrian scenarios. These events are represented using a novel and sophisticated load model. A comparison of the results highlights how (1) static deflections cannot be used to infer the maximum vertical accelerations of the bridge in service, and (2) certain pedestrian flows with pedestrian densities related to service describe stresses on different structural elements that are larger than those of the static design live loads, thus ULS combinations should also account for the dynamic performance of the different structural elements under pedestrian flows with pedestrian densities related to service.

PALABRAS CLAVE: Puente peatonal, factor amplificación dinámica, puente atirantado, cargas peatonales

1. Introduction

When designing structures under particular actions where loads are rapidly applied, such as railway bridges, highway bridges or footbridges, designers must evaluate the dynamic response of these structures when applying these loads. The evaluation of the dynamic structural response can be carried out performing a dynamic analysis, but it has also been traditionally been performed extrapolating this from static analyses applying dynamic amplification factors (DAF), if only the internal forces are required.

The use of DAF implies an exhaustive assessment of the dynamic response of the bridge under certain type of loads in order to define adequate values according to the phenomenon reproduced with these DAF.

When considering highway or railway bridges, several codes (*e.g.*, [1]) define the DAF to apply to static equivalent loads for the fatigue analysis at road bridges or the static equivalent railway loads (which depend on the degree of maintenance of the railway line), *etc.*

When looking at pedestrian bridges, there are methodologies for which the peak accelerations are inferred based on static deflections of the deck under live loads, without the need to perform dynamic analyses. The serviceability criteria is then checked by comparing these values with certain deflection limits. On the other hand, when evaluating aspects related to ULS, these factors are not usually described. Some codes or guidelines [1,2] imply that these DAF are already included in the static assessments with the proposed live loads, whereas others [3] highlight that the results of certain dynamic analyses must be compared to the static response.

This paper summarises the result of multiple numerical evaluations performed on a benchmark cable-stayed footbridge considering multiple pedestrian scenarios with the aim of describing the DAF that can be obtained when comparing deflections, bending moments and shear forces at different structural elements of this bridge under equivalent static and dynamic loads.

2. DAF for pedestrian bridges

Vertical pedestrian loads have been usually described mathematically using Fourier series, see Equation (1) (*e.g.*, [1,3]). Typically, this Fourier series has been truncated to one harmonic.

$$p(t) = W_p \left[1 + \sum_{n=1}^{\infty} b_n \sin(\omega_n t - \phi_n) \right] \tag{1}$$

where *p(t)* describes the total load introduced by a single pedestrian (both feet) when crossing over a structure, b_n are the harmonics of the Fourier series (typically only b_1 is defined) and w_n are the frequencies of each component (w_1 reflects the pedestrian vertical frequency).

The form of this pedestrian load representation is at least partly dictated by the available computational resources when it was first implemented and the simplicity to define deflections, speeds and acceleration of structures represented as single-degree-of-freedom (SDOF) systems under the action of harmonic loads (see [4]).

Due to the simple description of the deflection, speed and acceleration of a SDOF under harmonic loading based on the static deflection of the structure represented by the SDOF [4], several methods were put forward to predict the maximum serviceability accelerations of a footbridge based on such simplification. One of such methods corresponds to that of Blanchard *et al.* [5] published in 1977 and based on Equation (2). This method was implemented in the British Standard for bridge loads in 1978 [6].

$$a = 4\pi^2 \cdot f_1^2 \cdot y \cdot K \cdot \Psi \tag{2}$$

where a is the resulting peak acceleration (m/s^2) under the passage of a single pedestrian with a weight of 700N, f_1 is the fundamental vertical frequency of the bridge in Hz, y is the static deflection at mid-span when a 700N point load is applied at that point, K is a configuration factor that depends on the number of spans and Ψ is a dynamic response factor that depends on the main span length and the bridge damping.

In relation to the stresses to be used for the design of pedestrian bridges (ULS assessments), relevant publications and codes normally include in equivalent static live load magnitudes the effects of the dynamic loads in service:

- BS EN 1991-2:2003 [1] highlights that 5kN/m^2 already includes the dynamic effects of pedestrians and that peak accelerations must be derived from a dynamic analysis (loads to be defined in the annex of each country).

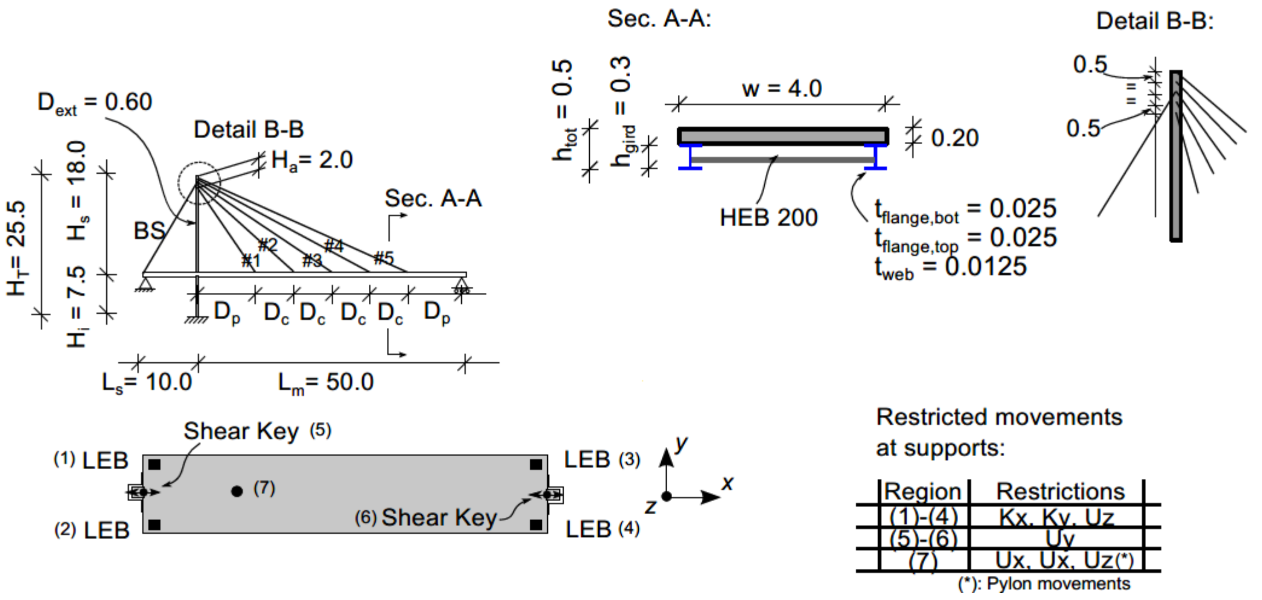

Figure 1. Geometric and structural characteristics of the benchmark cable-stayed footbridge; articulation of the footbridge deck. Dimensions in meters.

- fib Bulletin 32 [2] describes how the static loads for ULS analysis is sufficient to cover the dynamic effects of different pedestrian events in service.
- Nonetheless, Setra Guideline [3] proposes performing ULS analyses with the results of dynamic events (with damping ratios larger than those in service) as well as ULS analyses with results from large dynamic events (large demonstrations or vandal loads).

3. Numerical assessments

In order to assess the validity of proposals included in codes, guidelines and other relevant publications, the following section summarises the numerical results described by a cable-stayed bridge with moderate main span length (50m) under the effect of alternative pedestrian scenarios (low, medium and high pedestrian density).

3.1 Footbridge: geometry and dynamic characteristics

The cable-stayed footbridge considered for these numerical analyses has a geometrical and structural arrangement derived from the compilation of 38 cable-stayed bridges already built. Information of these bridges has been obtained from multiple sources (*e.g.*, [7]) and can be found summarised in [8].

Based on this database of cable-stayed footbridges already built, most footbridges with this structural typology have a main span with length between 50m and 100m (main span lengths near 150m are far less common), with average values near 50m. The geometric characteristics of a representative cable-stayed footbridge are as follows:

- Two spans: main span with length (L_m) of 50m and a side span (L_s) with length 0.20L_m.
- Composite deck consisting of a concrete slab supported by two steel girders. The database of real footbridges shows a certain variation (steel box girder, steel girders with decks of different materials and the proposed solution, slightly more common than the rest).
- Pylon consisting of a single vertical mono-pole element, with a steel circular hollow cross-section. Alternative pylon shapes such as ‘A’, inverted ‘Y’ or ‘H’ are less common for footbridges.
- The most common pylon height (and used for the benchmark structure)

Table 1. Vibration modes and frequencies [Hz] of the cable-stayed footbridge, where 'VN', 'TN' and 'LN' denote vertical, torsional and lateral modes with N half-waves in the main corresponding structural span (from the pylon to the abutment support section for vertical and torsional modes; and between abutment support sections for lateral modes), 'Ld' longitudinal and 'P' pylon modes.

Mode no.	Description	Empty	0.2 ped/m^2	0.6 ped/m^2	1.0 ped/m^2
1	V1	1.01	1.00	0.98	0.96
2	Ld.	1.14	1.13	1.11	1.09
3	P	1.21	1.20	1.20	1.20
4	V2	2.02	2.00	1.96	1.92
5	L1	2.23	2.21	2.16	2.12
6	T1	2.98	2.96	2.93	2.89
7	V3	3.28	3.25	3.18	3.12
8	T2	3.82	3.79	3.73	3.67
9	V4	5.08	5.03	4.92	4.83
10	T3	5.57	5.52	5.44	5.36
11	P	7.02	7.01	6.99	6.96
12	V5	7.44	7.36	7.20	7.06

corresponds to 0.36L_m.

- Deck depth equivalent to a span ratio of L_m/100 (*i.e.*, 0.50m).
- The cable system is arranged as a semi-fan configuration. Harp arrangements are less common for these bridges (due to their lower structural efficiency).
- The spacing between anchorages of the stay cables in the deck is chosen from the average value in the database (7m).
- The deck width is generally chosen according to the size of the pedestrian streams expected to use the footbridge. The database shows an average value of 4m and this is adopted to define the geometry of the representative cable-stayed footbridge.
- The bridge deck is supported by laminated elastomeric bearings (LEB) and a shear key in each abutment. The pylon restricts the relative movements of the deck but not its rotations.

The dimensions of the structural elements have been derived considering their performance under permanent and live loads. The geometry of the cable-stayed footbridge is represented in Figure (1).

The first vibration modes of this benchmark cable-stayed footbridge are summarised in Table (1). These have been defined using FE models developed with ABAQUS, a commercial software widely used in research analysis in the civil engineering field. In these models, the deck slab as well as the longitudinal steel girders have been represented using shell elements, the transverse steel girders as beam elements and the cables as single truss elements. Table (1) also summarises the effect of the mass of pedestrians in a flow on the magnitude of the first frequencies.

The results of Table (1) highlight that these bridges define vertical modes within the range considered critical (*e.g.*, 1.7Hz-2.1Hz according to [3]), whereas lateral and torsional modes are outside the critical range in each case. The pedestrian mass has a very moderate effect on the frequency of mode V1, and slightly larger on modes V2 or higher and on lateral and torsional modes.

3.2 Representation of pedestrian loads for dynamic analysis

When evaluating the dynamic response in service of any pedestrian bridge, codes and guidelines describe the characteristics of the pedestrian serviceability events according to the likelihood of coincidence of vibration modes of the bridge (and harmonics) with the step frequencies commonly used by pedestrians. As an alternative to this procedure, the evaluation of the dynamic response in service of cable-stayed footbridges has been performed using a pedestrian loads model developed by the authors based on existing literature in different fields. This pedestrian load model does not depend on the vibration characteristics of the structure and is based on the representation of the vertical and horizontal loads individually generated by each pedestrian at each step on the bridge, as well as on the intra-subject variability (related to the inability of individual subjects to repeat monotonous activities with constant features such as step length or step frequency), inter-subject variability (related to the wide range of characteristics within the human population) and on the effect of pedestrians on others (due to the density of the pedestrian flow). The load model is based on empirical data obtained by [9], on the work of [10-12] for lateral loads and of [13] for the representation of pedestrian flows. A detailed description of this pedestrian load model can be found in [8].

Regarding the fundamental characteristics of the pedestrian flows to be considered for the dynamic analyses of the benchmark cable-stayed bridge, several pedestrian events have been considered in order to capture the effect of possible alternative locations of such a bridge. According to that, the pedestrian flows numerically simulated include low, medium and high pedestrian densities (0.2, 0.6 o 1.0 pedestrians per deck square meter, 'ped/m^2'). Pedestrians of these flows may be commuting or travelling related to leisure, and this has an impact on speed and step frequencies used by the pedestrians, see [8].

Table 2. Peak vertical and lateral accelerations [m/s^2] generated by each pedestrian flow simulated on the benchmark cable-stayed footbridge, where 'C' corresponds to pedestrian flows while commuting and y 'L' while travelling for leisure.

Method	0.2 ped/m^2	0.6 ped/m^2	1.0 ped/m^2
Vertical accelerations	1.16 (C) / 0.81 (L)	1.70 (C) / 1.62 (L)	2.15 (C) / 2.07 (L)
Lateral accelerations	0.16 (C) / 0.10 (L)	0.31 (C) / 0.18 (L)	0.46 (C) / 0.40 (L)

The maximum vertical and lateral accelerations generated by each pedestrian flow are summarised in Table (2). More details regarding the magnitude of the accelerations and its distribution along the length of the deck in each case can be found in [8,14].

4. Dynamic amplification factors for cable-stayed footbridges

4.1 Dynamic amplification factors related to deflections

As discussed in Section 2, there are several methods published in relevant literature that provide an assessment of the peak vertical accelerations of pedestrian bridges in service based on the static deflections of this footbridge [5]. As an alternative, other authors propose limiting the maximum static deflections of the bridge in order to ensure an adequate dynamic response of the bridge in service [15].

The maximum dynamic deflections of the cable-stayed footbridge are described in Figures 2-3.

In the vertical direction, Figure 2 highlights maximum dynamic deflections equivalent to $L_m/3676$, $L_m/1689$ o $L_m/1101$ for pedestrian flows with low, medium and large pedestrian densities. These dynamic deflections are equivalent to DAFs for deflections ranging from 2.27 to 1.49 (see Figure 4, intermediate plot, for the comparison of the results for serviceability events with multiple pedestrian densities).

The results for the dynamic vertical deflections highlight several aspects:

1. The maximum dynamic deflections are produced at a distance of 45m from the left abutment (35m from the pylon, i.e., 70% of the span). However, the peak vertical accelerations are described at sections located 28m from the same abutment, near an antinode of the mode V2.
1. The dynamic deflections do not depend on the type of pedestrian flow (for medium and high density pedestrian flows), despite the fact that vertical accelerations depend on the type of flow (see Table 2).

From the previous points it is derived that

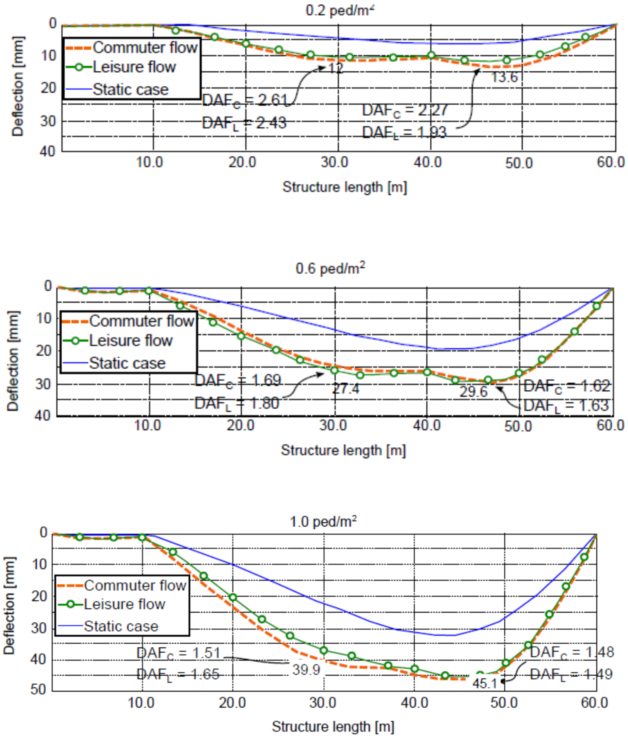

Figure 2. Dynamic and equivalent static vertical deflections caused by pedestrian flows with 0.2 ped/m² (upper figure), 0.6 ped/m² (middle figure) y 1.0 ped/m² (lower figure).

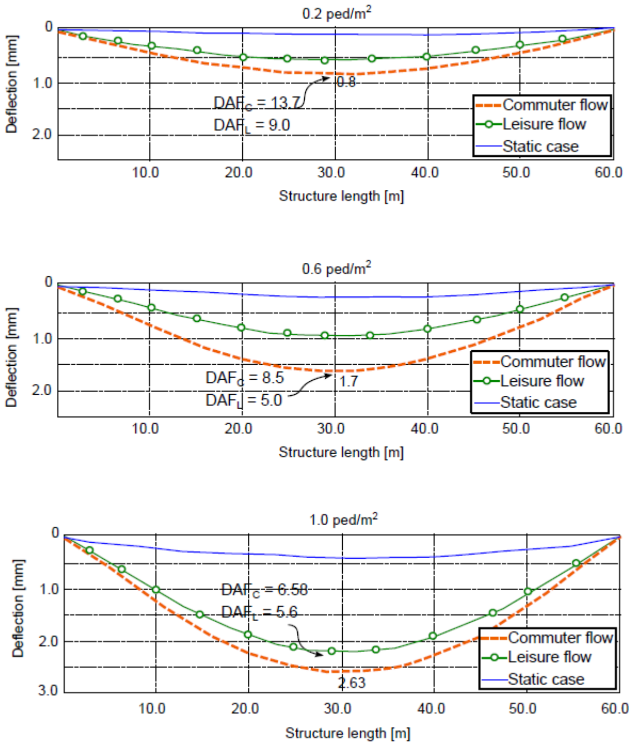

Figure 3. Dynamic and equivalent static lateral deflections caused by pedestrian flows with 0.2 ped/m² (upper figure), 0.6 ped/m² (middle figure) y 1.0 ped/m² (lower figure).

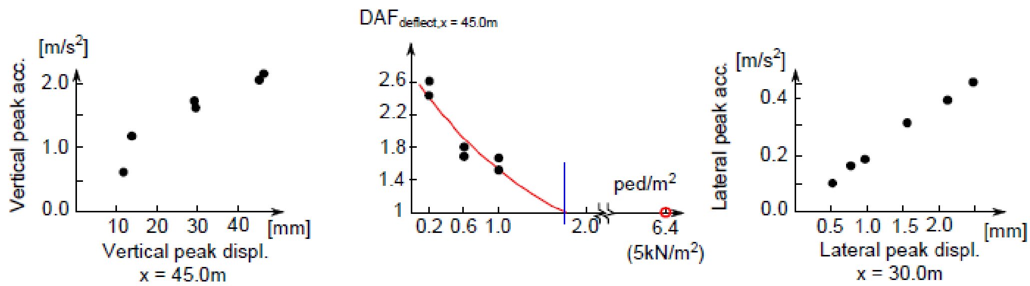

Figure 4. Comparison between vertical peak deflections and vertical peak accelerations described at the same events (left); DAFs related to vertical deflections at x = 45.0m and pedestrian flow density causing these dynamic deflections (middle); and comparison between lateral peak deflections and lateral peak accelerations at the same events (right).

vertical accelerations and static deflections are not well correlated and thus the definition of the former based on the latter may be inaccurate and unrealistic. This phenomenon in this bridge is related to the contribution of torsional modes (with vertical and lateral components), being activated by the lateral loads that end up enhancing the vertical movements. This effect cannot be captured considering the deflections of the deck under exclusively vertical loads.

When looking at the relation between number of pedestrians in the flow and the DAF related to vertical deflections at x = 45m (see Figure 4, centre) it can be seen how the extrapolation of results shows that for pedestrian flows larger than 2 ped/m² the dynamic component is almost negligible and the DAFs tend to one. This may be caused by the larger number of interactions between pedestrian and the reduced walking speeds and step frequencies of each user. Nonetheless, it should be expected that pedestrian flows with 2 ped/m^2 cause vertical accelerations larger than zero (otherwise pedestrians might be practically stopped on the bridge deck).

In lateral direction, Figure 3 represents the maximum dynamic deflections obtained in the same pedestrian dynamic events. The dynamic deflections have values ranging from 0.5mm to 2.6mm, well below the 10mm described by [16] as limiting value that would disturb pedestrians on the deck and generate a change in their step frequency and step to adapt themselves to the movements of the bridge, phenomenon known as pedestrian synchronisation with the lateral movements of the bridge.

The evaluation of the DAFs for the lateral deflections of Figure 3 shows how there is no clear decrease of these DAFs when increasing the number of pedestrians of the flow on the bridge.

From Figures 3 and right plot of Figure 4, it can be inferred how, as opposed to the vertical movements, the lateral deflections depend on the type of pedestrian flow that crosses the bridge. Furthermore, there is a positive and clear linear correlation between the number of pedestrians on the deck and the maximum lateral acceleration, *i.e.*, the lateral dynamic response is linearly related to the number of pedestrians crossing the bridge and their walking speed.

In this lateral direction, the static deflections (with load magnitudes as defined in [1]) would not be enough to predict the peak lateral accelerations generated by pedestrians.

In summary, for a bridge with this structural type, both vertical and lateral dynamic deflections or accelerations cannot be predicted based on static analyses of the live loads deflections

4.2 Dynamic amplification factors related to ULS (bending moments and shear forces) of the deck

Figure 5 represents the magnitudes of the

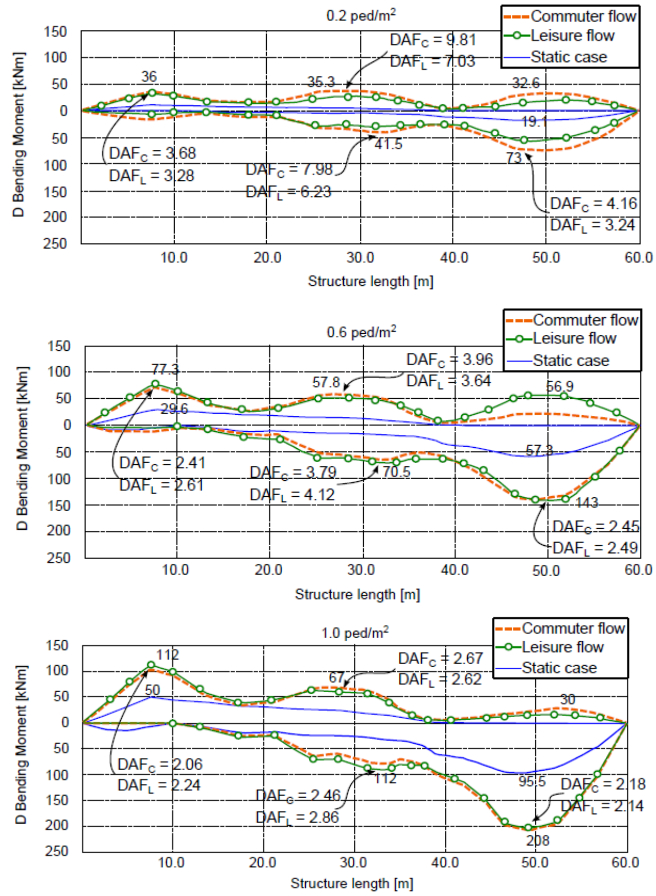

Figure 5. Dynamic and equivalent static bending moments caused by pedestrian flows with 0.2 ped/m² (upper plot), 0.6 ped/m² (middle plot) and 1.0 ped/m² (lower plot).

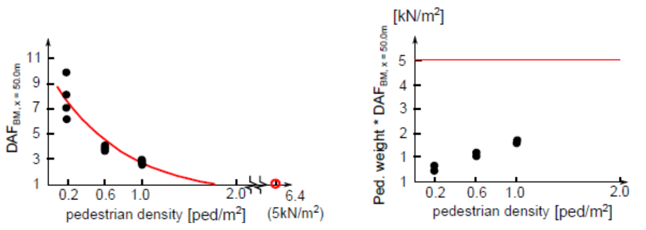

Figure 6. Comparison between DAFs related to deck bending moments at x = 50.0m and pedestrian traffic density causing these dynamic stresses (left) and similar correlation considering the weight of the traffic flow compared to the weight of the ULS live load.

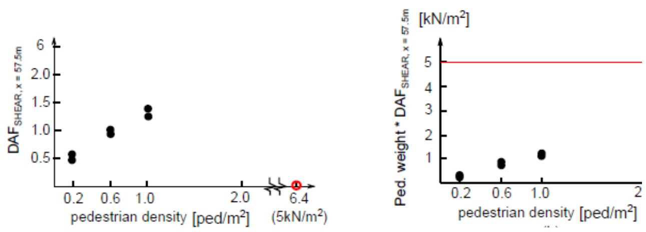

Figure 7. Comparison between DAFs related to shear forces at x = 57.5m and pedestrian traffic density causing these dynamic stresses (left), and similar comparison considering the weight of the traffic flow (right).

bending moments generated by the different serviceability pedestrian events numerically simulated and the magnitudes of these bending moments relative to the static bending moments generated at the same sections by the static equivalent weight of the simulated pedestrian flows.

This comparison between static and dynamic bending moments highlights how the largest difference between both magnitudes at a same section is given at x = 50.0m, at the main span near the abutment. At this section, static live loads do not generate hogging bending moments, as opposed to what dynamic loads generate (as a result of the shape of the vibration modes that trigger the movements).

The difference between bending moments generated with the static loads or the equivalent dynamic loads is fairly large along the whole length of the bridge, in particular when the flows have a small number of pedestrians. Nonetheless, both static and dynamic bending moments have a moderate magnitude, as seen in Figure 5 (upper plot).

Similarly to vertical accelerations, the magnitudes of the dynamic deck bending moments depend on the type of pedestrian flows for small flow densities. However, the type of flow does not have an effect for densities similar or higher than 0.6 ped/m^2.

Figure 5 also describes how the section with largest dynamic bending moments is that located at x = 50.0m. However, the section with largest DAF related to these bending moments is produced at a section located at x = 30.0m (disregarding the sections with dynamic bending moments with sign not described under static equivalent loads).

According to Figure 5, the events with a large number of pedestrians (1.0 ped/m^2) describe maximum DAFs for bending moments lower than 2.86 at the critical sections. Accordingly, a series of static analysis with equivalent weights of 2.86 ped/m^2 (2.25kN/m^2) or larger would give place to bending moments similar to those of the static loads (see Figure 6). However, this would not be valid for the hogging bending moments near x = 40m.

For moderate flow densities, the DAFs related to bending moments of the deck are larger than those defined with higher pedestrian flows. This has also been observed for the vertical dynamic deflections described in the previous section.

The DAFs related to deck bending moments are larger than the DAFs related to the deck deflections of the same events. This difference is justified by the number of modes that participated in the structural response of the bridge (see [17]). If the dynamic response had been generated by a single vibration mode, both DAFs would have had more similar values. However, several modes participate in the dynamic response of the footbridge and, accordingly, the DAFs related to bending moments are higher than those for the deflections of the same structure (see [17]).

A similar analysis of the shear forces of the deck during the dynamic events emphasises that these dynamic shear forces are similar or considerably lower than the static shear forces at the deck (see Figure 7). The largest difference between static and dynamic forces is produced near the abutment at x = 60.0m.

The DAFs for deck shear forces have values near 0.5 for small density flows and 1.4 for larger flows. As opposed to deflections and bending moments, the magnitudes of these DAFs increase with the density of the flow. This effect is due to the fact that the dynamic structural response is basically generated by the first modes of vibration of the structure and DAFs could only have larger values with the contribution of higher frequency modes (see [17] for a justification regarding this phenomenon).

4.3 Dynamic amplification factors related to ULS of the pylon

The static stresses generated by the pedestrian live loads on the pylon of the cable-stayed bridge have very small magnitudes. Despite that, the different dynamic serviceability event numerically reproduced generate considerably larger stresses at the pylon.

The axial loads supported by the pylon are between 1.05 and 1.60 times larger than the static stresses reproduced by the equivalent static scenarios.

The DAFs related to the bending moments at different sections of the pylon are considerably larger than those for axial loads. These range between 3.4 and 7.6 at the support of the pylon and between 2.6 and 4.1 at the section near the deck. At the section near the deck specifically, the comparison between these DAFs and the densities of different flows that generate these DAFs highlight how dynamic and static loads would only cause similar effects for pedestrian flows with densities beyond 8.0 ped/m^2. Accordingly, the ULS calculations based on bending moments at the pylon using results considering 5kN/m^2 (equivalent to 6.4 ped/m^2) do not cover the dynamic effects produced by large flows of pedestrians in this case.

These large dynamic effects are explained by the vibration modes that generate the dynamic response. Results show that vertical and lateral accelerations of the bridge are participated by deck torsional modes. Thus, not only vertical loads generate vertical movements but also lateral pedestrian loads.

Thus, ULS calculations using 5kN/m^2 should always be complemented with ULS calculations using values derived from dynamic analyses.

5. Conclusions

The dynamic amplification factors (DAF) are commonly used for the design of railway and highway bridges. For pedestrian bridges, DAFs have been used to provide a simplified assessment of the peak vertical accelerations in service of the structure. Alternatively, codes and guidelines consider the use of static live loads of 5kN/m^2 includes the effect of any pedestrian scenario to design for ULS.

This paper evaluates the simplifications mentioned in the previous paragraph through multiple numerical analyses of a cable-stayed footbridge that can be considered as representative of this structural typology. According to these numerical analyses it can be seen that:

1. The vertical static deflections cannot be used to predict the maximum dynamic response. The dynamic deflections of the deck in service are not correlated to the magnitudes of the accelerations. This is due to the number of modes of vibration that participate in the dynamic response.
2. The static analysis using live loads of 5kN/m^2 do not cover all the dynamic stresses generated by different pedestrian flows (different densities of pedestrians on the deck). Accordingly, ULS should be assessed from results obtained in static analyses and results obtained from a set of possible dynamic pedestrian events.

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