¿Es necesario considerar las deformaciones impuestas en Estado Límite Último?

Imposed strains in Ultimate Limit State: do we need to consider them?

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RESUMEN

En este trabajo se estudia la necesidad de considerar los efectos de las deformaciones impuestas en Estado Límite Último (ELU) en pilares sometidos a distintos grados de fuerza axil. El estudio se basa en la determinación de la fuerza horizontal última que puede absorber un sistema formado por dos pilares unidos mediante un tirante sometido a valores crecientes de deformación de acortamiento. Se demuestra que para obtener valores realistas de la pérdida de capacidad por el efecto de las deformaciones impuestas es necesario considerar los efectos de confinamiento del hormigón por la armadura transversal y el efecto del decalaje de la ley de momentos por efecto de la fisuración diagonal a cortante en la base del soporte.

ABSTRACT

This paper explores the need to account for the effects of imposed strains in ULS, in reinforced concrete columns subjected to varying degrees of axial force. The study is based on the determination of the ultimate horizontal force of a pair of columns linked by a tie on which varying values of compressive imposed strain is applied. The study considers a slender column with a small amount of reinforcement and a stocky column with a high amount of reinforcement. It is shown that, in order to obtain a realistic evaluation of the loss of capacity, it is necessary to account for confinement of concrete by the stirrups and the effect of the shift in the bending moment law due to cracking of the section in shear.

PALABRAS CLAVE: deformaciones impuestas, ductilidad., ELU, hormigón confinado.
KEYWORDS: imposed strains, ductility, ULS, confined concrete

1. Introduction

Modern concrete Standards such as Eurocode 2 (EN 1992-1-1:2004, CEN (2004)) [1] allow to neglect the effect of imposed deformations in ULS ‘provided that the ductility and rotation capacity of the elements are sufficient’. This provision is of great importance to simplify design while avoiding gross over-dimensioning which would result from the consideration of imposed strains using linear elastic analysis with gross cross section properties. However, the need to satisfy the ductility requirement remains vague. For beam elements with no axial load, analysis is simpler and there is both theoretical [2] and experimental [3] research substantiating this statement and providing ductility criteria. However, the problem is not as well documented for supports with varying degrees of axial force. This aspect becomes critical when imposed strains become a significant action as is the case of long jointless structures.

Focusing on the design of supports, the purpose of this paper is to study what the effect of imposed strains really is in ULS. The assumption is that the support is subjected to a quasi-permanent level of axial force, which may vary between 40 and 60% of the compressive capacity of the concrete section and the effects of a growing value of imposed displacement and that it will fail due to the applied horizontal forces. The analysis is meant to determine the loss of ultimate bearing capacity, expressed as the maximum horizontal force that can be resisted, as the imposed strain increases.

A comprehensive study should account for the following design parameters:

- ✓ Support slenderness as quantified by d/L^2
- ✓ Level of quasi-permanent axial force (ν_{qp})
- ✓ Longitudinal reinforcement ratio ($\rho_l=A_s/A_c$)
- ✓ Transversal confinement reinforcement ratio (ρ_w)
- ✓ Short-term and long-term imposed strains.

Due to space limitations, in this paper only two practical cases, one a slender element with low reinforcement, and the other a stockier element with a high reinforcement ratio are analysed. Both supports are analysed for three levels of axial forces and 3 levels of confinement reinforcement. This analysis is carried out only for short-term imposed strains.

2. Methodology

2.1 Procedure

To evaluate the effects of imposed strains a very simple scheme has been devised in which two cantilevered columns are united by a hinged tie (see Fig. 1). Each column is subjected to an axial force representative of the quasi-permanent load combination. An imposed strain is applied on this tie to achieve a given displacement at the top of the column. Then a horizontal force is applied on the structure at the top of the columns. If the strain is negative (contraction) the previous stress state due to the imposed strain will be favourable for column whose displacement is contrary to the direction of the force (Column 2) and unfavourable for the companion column (Column 1). The analysis consists in determining to what extent the column which is more severely loaded, and which will reach its maximum capacity first will be able to deform without reaching its ultimate strain plane so that the other column can, itself reach the ultimate capacity.

The structural scheme represented in Fig. 1 would also simulate the behaviour of a doubly embedded column of twice the height.

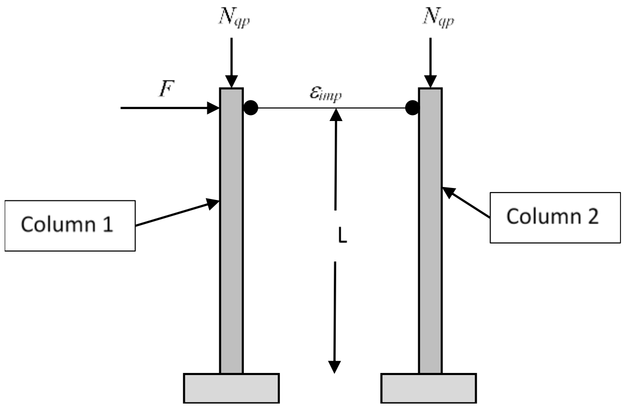


Figure 1. Structural scheme considered for the study.

The methodology used to determine the behaviour of the column pair when subjected first to an imposed displacement and then to a horizontal force is the following:

- ✓ For a given axial load N_{qp}, determine the load-deflection diagram of a single column, $F(\delta)$, by pushover analysis. Determine the value of the maximum displacement, δ_{max}.
- ✓ The maximum horizontal force that can be carried by Column 1 will be $F(\delta_{max})$- $F(\delta_0)$.
- ✓ Then, for each value of the imposed displacement δ_0,

- Determine the load on the column that is needed to produce this displacement, $F_0=F(\delta_0)$.
- Determine the remaining displacement that can be taken by the column as $\delta_{remaining} = \delta_{max}-\delta_0$.
- If $\delta_{remaining} < \delta_0$ then the maximum force that can be taken by Column 2, without previous failure of Column 1, F_2, will be equal to $F_2=F(\delta_0)-F(\delta_0-\delta_{remaining})$. If not, $F_2=F(\delta_0)+F(\delta_{remaining}-\delta_0)$

It is most likely that the column will crack in combined flexure and shear, at least at the embedment. In this case the push-over analysis can be modified to account for the shifting of the bending moment law as shown in Fig 2. This effect is very significant because of the small slope of the moment curvature diagram when the load is reaching its maximum value. The approach shown in Fig. 2 will slightly overestimate the horizontal displacement because not all the sections will be equally cracked. However, the largest part of the deflection will be due to the curvature at the bottom of the cantilever because it has the largest lever arm and at ULS the non-linearity of the moment-curvature diagram will make this curvature much greater than the curvature of adjacent sections (Fig.2 underestimates the horizontal scale, due to space limitations). In fact, good and slightly conservative estimate of the deflection at the top of the cantilever would be $\delta=(1/r)_u \times d \times L$, where $(1/r)_u$ is the curvature corresponding to the maximum load.

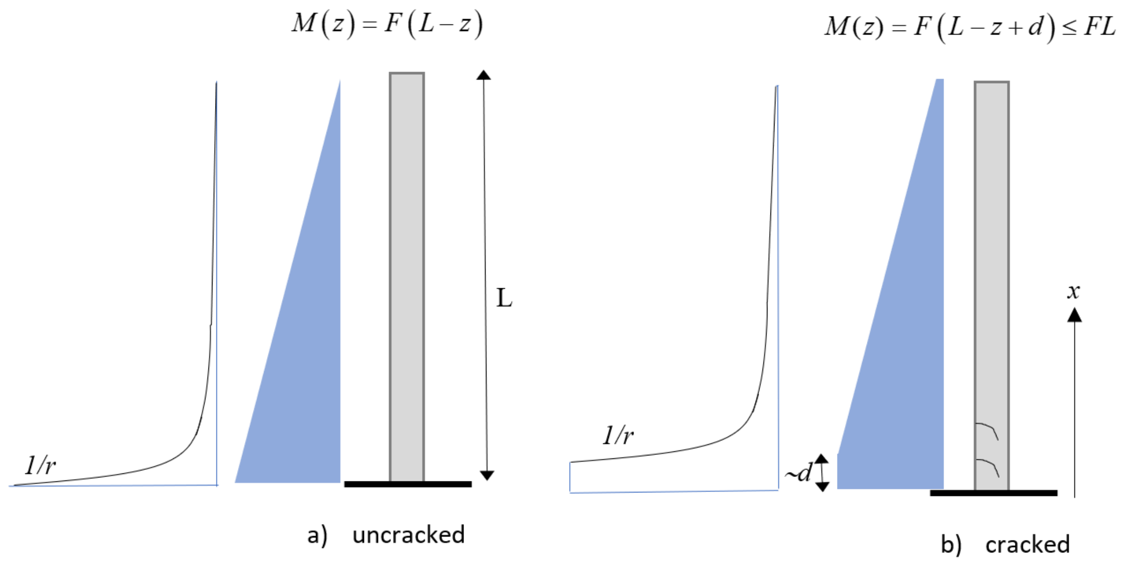


Figure 2. Shift of bending moment law due to cracking in combined flexure and shear

When the moment-curvature diagram is non-monotonous, which can happen when the constitutive law of concrete has a softening behaviour, it is not possible, in the load-deflection curve or pushover diagram to enter the softening zone. This is because the moment has a triangular distribution and is determined by equilibrium. When the maximum load is reached, going beyond this point would require a reduction in the moment in all sections. This means that the section right next to the embedment will never reach a curvature that is higher than the curvature corresponding to the maximum load. Since there must be continuity in the curvatures, it is not physically possible to take advantage of the softening branch and transfer more load to Column 2 of Fig. 1) by unloading Column 1.

2.2. Safety Format

In order to determine the ultimate load of the two-column set-up as a function of the imposed displacement and be able to compare it with its ULS load without imposed displacement, nonlinear analysis is carried out. The safety format adopted for this analysis is the Global Resistance Method, in its simplified approach known as ECOV (See Ref . [4]). This method consists in estimating the value of the coefficient of variation of the resistance V_R by expression (1), where R_m is the resistance of the structure considering mean material properties and is R_k is the resistance of the structure considering characteristic material properties

$$V_R = \frac{1}{1.65}\ln\left(\frac{R_m}{R_k}\right) \quad \textbf{(1)}$$

This approach is sound if it is verified that the failure mode does not change in the two calculations.

The global resistance factor is determined according to (2), where α_R is sensitivity factor for the reliability of resistance and is taken as 0.8 and β is the reliability index which is taken equal to 3.8.

$$\gamma_R^* = e^{\alpha_R \beta V_R} \quad \textbf{(2)}$$

The design resistance is determined according to (3).

$$R_d = \frac{R_m}{\gamma_R^* \gamma_{Rd}} \quad \textbf{(3)}$$

The model uncertainty factor, γ_{Rd}, is taken as 1.06 as suggested in reference [5].

2.3 Confined concrete

Confined concrete is considered according to Mander's model [6]. The formulation introduced in EN 1998-2 [7] provides the mean confined material response. For this, a value of f_{cm}=25+8=33 MPa and f_{ym}=500×1.15=575 MPa are used. In order to apply the ECOV method, a characteristic response is also needed. This is obtained by using the same formulations with characteristic values.

2.4 Time dependent effects

Creep relaxation is not considered in the numerical applications that follow due to space limitations. To account for creep relaxation for long-term analysis a simple procedure is to multiply the strains of the concrete constitutive law by a factor of $(1+\varphi)$, where φ is the creep coefficient.

3. Results

Two cross-sections were analysed, one representing a small support with a low reinforcement ratio (25x25 reinforced with 4ϕ12) and one representing a large support with a relatively high reinforcement ratio (80x80 reinforced with 28ϕ25). The height of the supports was assumed to be 1.5 m. equivalent to a doubly embedded support 3 m high. The degree of confinement was varied by varying the spacing between the stirrups from 200 mm, to 100 mm to 50 mm. The stirrup diameter was assumed to be constant and equal to 12 mm. The normal rule for engaging bars has been followed, engaging one out of two bars if the bar spacing is less than 150 mm, and engaging all longitudinal bars if it is more. In all cases cover to the centreline of the longitudinal bars has been assumed as 5 cm. Regarding the material classes, concrete class C25/30 and steel B 500C according to EN 1992-1-1 have been assumed. Table 1 provides values of the longitudinal and transversal reinforcement ratios of both sections. The transversal reinforcement ratio is referred to the area of the confined concrete core.

Table 1. Longitudinal and transversal reinforcement ratios of studied sections.

Section	ρ_l (%)	Stirrup spacing (mm)	ρ_w (%)
25x25	0.72	200	0.61
		100	1.22
		50	2.43
80x80	2.15	200	0.38
		100	0.75
		50	1.51

3.1 Section with small area and low reinforcement ratio 25x25-4ϕ12

Table 2 and Fig. 3 illustrate the constitutive laws used for concrete to analyse the sections depending on the level of confinement. A significant gain in concrete strength, but more remarkably, in deformation capacity is achieved, even for a very low amount of transversal reinforcement.

Table 2. Main parameters of confined concrete.

ρ_w (%)		$f_{c,c}$ (MPa)	$\varepsilon_{c1,c}$ (‰)	$\varepsilon_{c,cu}$ (‰)
0.61	Mean	35.8	2.86	24.48
	Char.	26.9	2.75	27.75
1.22	Mean	45.8	5.87	36.08
	Char.	33.6	5.42	42.06
2.43	Mean	62.7	10.99	42.06
	Char.	42.5	10.08	50.88

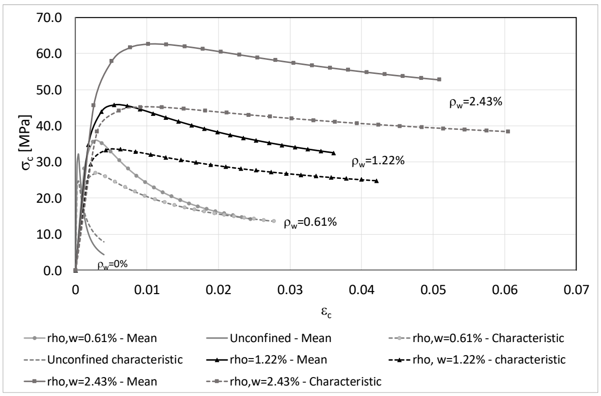


Figure 3. Mander diagrams for section 25x25

Figure 4 shows the parametric study for the support having a side dimension of 0.25 m. The figures show the ratio between the horizontal capacity of the column pair for a given imposed displacement to the capacity of the unstrained pair as a function of the inter-storey drift (displacement at the top of the column with respect to the bottom divided by the length of the support). Calculations are made for three different levels of axial force, equal to 40%, 50% and 60% of the axial capacity of concrete, which is determined as $f_{cd} \times b \times h$, where f_{cd} is the design strength of concrete taken as the characteristic strength f_{ck} divided by a partial material factor of 1.5 and for four levels of transversal reinforcement ratio, corresponding to no stirrups and stirrup spacings of 200, 100, and 50 mm.

Two calculations are made for each case, one in which diagonal cracking at the base is ignored (on the left) and one in which diagonal cracking is accounted for (on the right). Figure 4 shows that the horizontal capacity is reduced as the axial force increases and that the effect of diagonal cracking is quite significant.

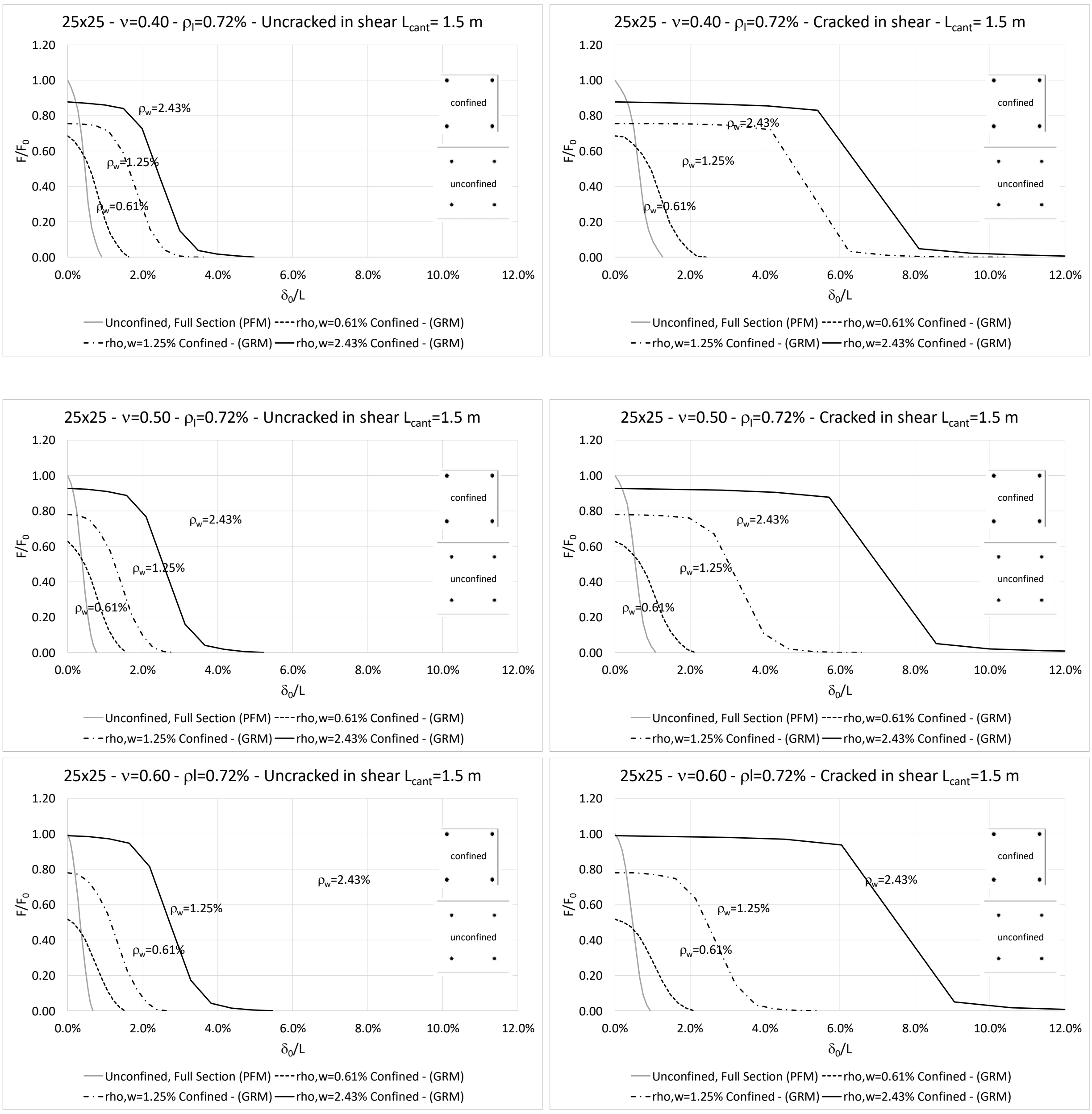


Figure 4. Resistance ratio as a function of inter-storey drift – left, assuming no shift rule and right, assuming shift rule

In most of the cases the capacity of the unconfined column pair is larger than that of the confined pair due to the reduction in the cross section in the confined sections due to spalling of the concrete cover. Of course, the capacity of the confined element should always be taken as the largest of the confined and the unconfined values.

Regarding the value of inter-storey ratio that can occur, 1% is an upper limit for seismic design according to EN 1998-1-1 [8]. This value can be taken as a reference. For a 3-meter doubly embedded column, this would be a 30 mm displacement. This can be achieved if the stirrup spacing is set to 50 mm, but not for a larger spacing, mainly because of the loss of capacity due to the spalling of the cover. However, with a spacing of 100 mm, it is still possible to keep 80% of the horizontal capacity.

In terms of imposed strains in a long concrete frame, it can be assumed that roughly 0.2 mm/m can be absorbed by cracking of the concrete floor (see [8]), which will tend to extend the length of the beam. Due to temperature, around 0.2 mm/m can be added. This effect will more or less cancel the effect from cracking of concrete. Regarding creep and shrinkage, assuming a creep coefficient of 2, a shrinkage strain of 0.5 mm/m and that the quasi-permanent maximum strain at the critical sections is $0.4\times f_{ck}$,, the mean applied strain can be roughly estimated by the expression of Eq. (4) (see [9]), where the factor 0.5 is meant to average the concrete stress over the span:

$$\varepsilon_t \sim \left(\varepsilon_{cs} + \varphi \times 0.5 \times 0.4 \frac{f_{ck}}{E_c}\right)\underbrace{\left(1-\frac{h}{2d}\right)}_{\approx 0.4} = \qquad \mathbf{(4)}$$

$$= \left(0.5 + 2 \times 0.5 \times 0.4 \frac{25}{30000}\right) \times 0.4 = 0.20 \text{ mm/m}$$

Assuming that for long-term analysis a 30% larger displacement can be accommodated due to creep relaxation, so that the inter-storey drift can be 1.3%, the length of the structure, from the point of zero-movement to its end with maximum displacement, could be around 0.013/0.00020×L~195 m (L=3), so that a symmetrical structure could have a length of up to 390 m. This number is very high. However, for a 25x25 column it can only be reached if the columns are properly confined (stirrup spacing equal to 50 mm or equivalent measure).

3.2 Section with large area and high reinforcement ratio 80x80-28ϕ25

Table 3. Main parameters of confined concrete.

ρ_w (%)		***fc,c*** **(MPa)**	$\varepsilon_{c1,c}$ **(‰)**	$\varepsilon_{c,cu}$ **(‰)**
0.38	Mean	41.7	4.63	14.93
	Char.	30.8	4.32	16.87
0.75	Mean	51.2	7.50	21.82
	Char.	37.2	6.89	25.29
1.51	Mean	66.0	12.00	31.63
	Char.	47.5	11.02	37.35

Table 3 and Fig. 5 illustrate the constitutive laws used for concrete to analyse the 80x80 section depending on the level of confinement. Even though the transversal reinforcement ratios are smaller than before the improvement of the concrete properties are similar to those obtained for the smaller cross section.

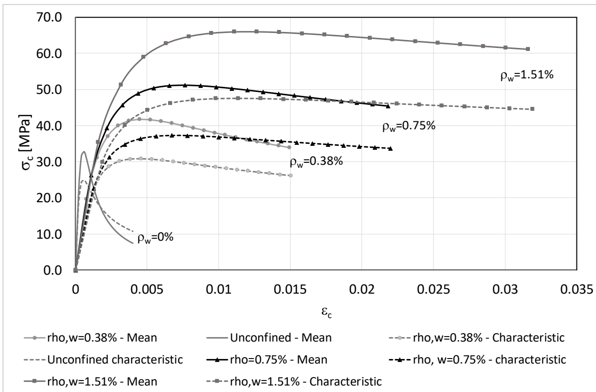
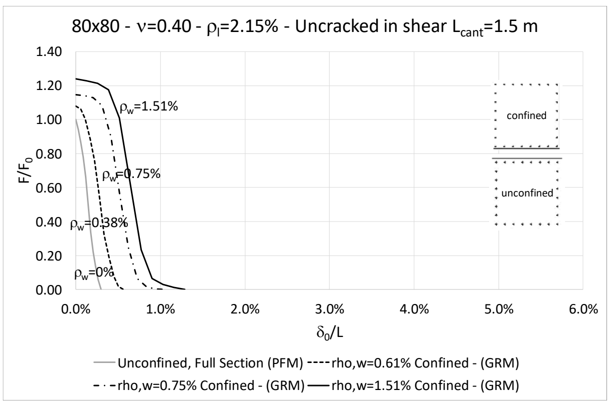
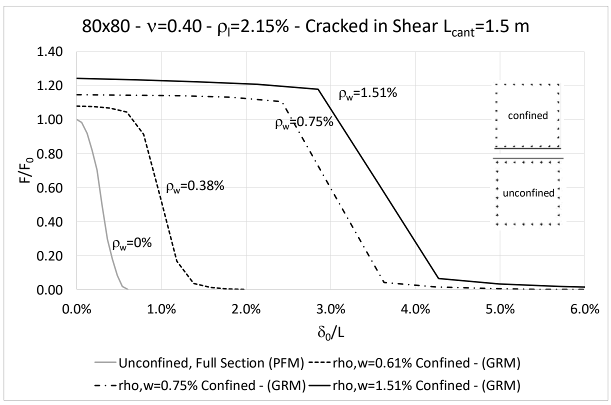


Figure 5. Mander diagrams for section 25x25





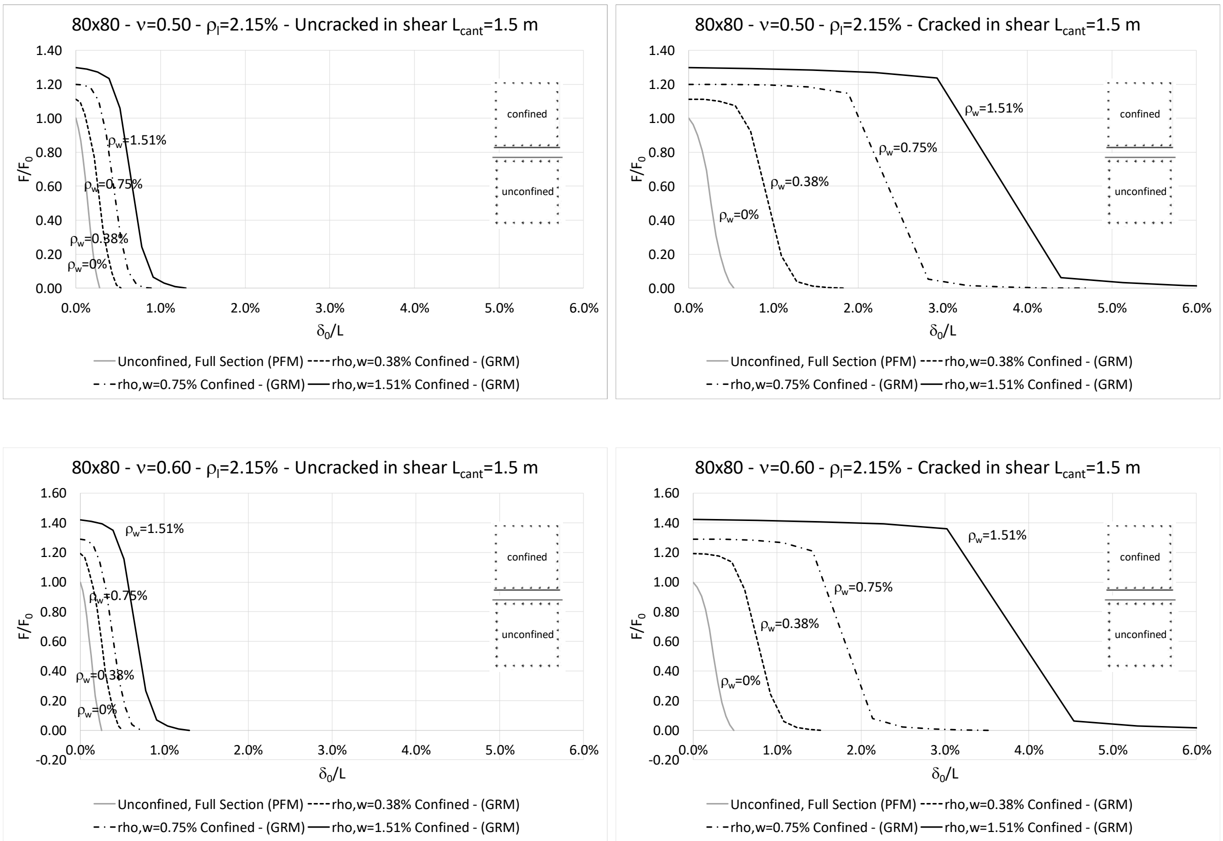


Figure 6. Resistance ratio as a function of inter-storey drift – left, assuming no shift rule and right, assuming shift rule

For the 80x80 section there are some differences with respect to the 25x25 section (see Fig. 6). In this case, the effect of the shift rule is more significant. Accounting for this effect, having stirrups with a 100 mm spacing is enough to accommodate inter-storey drifts significantly higher than 1% without loss of horizontal capacity. Also, in this case, the capacity obtained by considering the confined sections is always higher than the resistance of the unconfined section. This is because the section loss due to the spalling of the cover is much smaller in relative terms (88% for 80x80 and 55% for 25x25) .

4. Conclusions

From the above considerations the following conclusions can be drawn:

- ✓ ULS accounting for the effects of imposed strains should be checked in long jointless structures. In order to obtain realistic results, account must be taken of confining reinforcement and of diagonal cracking due to shear, especially for stocky sections.
- ✓ When designing long jointless structures, consideration should be given to providing confinement reinforcement in order to ensure adequate compliance with ULS.
- ✓ The resistance of small sections can be significantly reduced due to spalling of the cover for large imposed displacements. In such cases a significant confinement reinforcement may be needed.

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